Math 163A Handout 6: Example of Graphing a Polynomial with the 10-Step Method

Use the 10-step method to produce a graph of the function $f(x)=x^3-9x^2+15x-7$.

You may use the following information: $\begin{cases} f(x)=x^3-9x^2+15x-7=\left[(x-1)^2\right](x-7) \\ f'(x)=3x^2-18x+15=3(x-1)(x-5) \\ f''(x)=6x-18=6(x-3) \end{cases}$

Solution

Step 1: $f(0)=(0)^3-9(0)^2+15(0)-7=-7$, so the point $(0,-7)$ is the *y*-intercept.

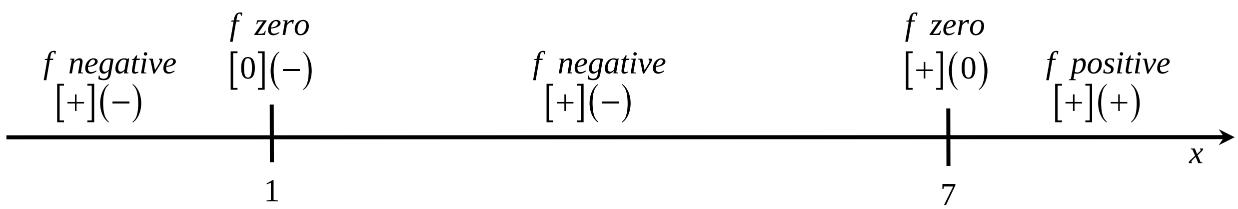
Step 2: To check for symmetries, we compare the three functions $f(x)$, $f(-x)$, and $-f(-x)$.

$$\begin{cases} f(x)=x^3-9x^2+15x-7 \\ f(-x)=(-x)^3-9(-x)^2+15(-x)-7=-x^3-9x^2-15x-7 \\ -f(-x)=-\left(-x^3-9x^2-15x-7\right)=x^3+9x^2+15x+7 \end{cases}$$

Because none of these match we conclude that the graph of f will not have *y-axis or origin symmetry*.

Step 3: The end behavior will resemble $y=x^3$. That is, "down on the left, up on the right.

Step 4: $f(x)=x^3-9x^2+15x-7=\left[(x-1)^2\right](x-7)$. The sign chart for f is:

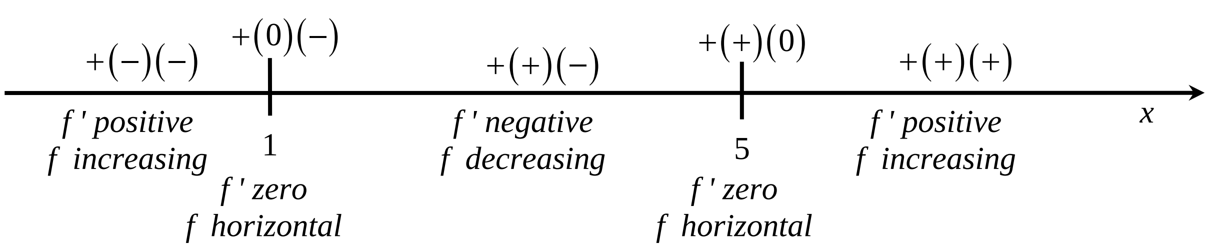


sign chart for $f(x)=x^3-9x^2+15x-7=\left[(x-1)^2\right](x-7)$

Step 5:

- The linear factor $(x-1)$ appears in the numerator and not the denominator. Therefore, step 5 case 1 applies: the graph of f will have an *x*-intercept at $x=1$. (This is confirmed by the sign chart. Good.)
- The linear factor $(x-7)$ appears in the numerator and not the denominator. Therefore, step 5 case 1 applies: the graph of f will have an *x*-intercept at $x=7$. (This is confirmed by the sign chart. Good.)

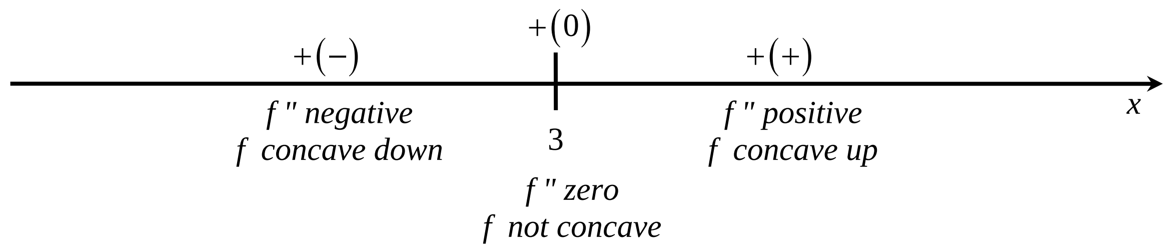
Step 6 & 7: $f'(x)=3x^2-18x+15=3(x-1)(x-5)$. The sign chart for f' is:



sign chart for $f'(x)=3x^2-18x+15=3(x-1)(x-5)$

We see that the numbers $x=1$ and $x=5$ are critical numbers for f. We already know that the point $(1,0)$ is on the graph. But we still need to find the y-value corresponding to $x=5$. We find that $f(5)=-32$. Therefore, there will be a critical point at $(5,-32)$.

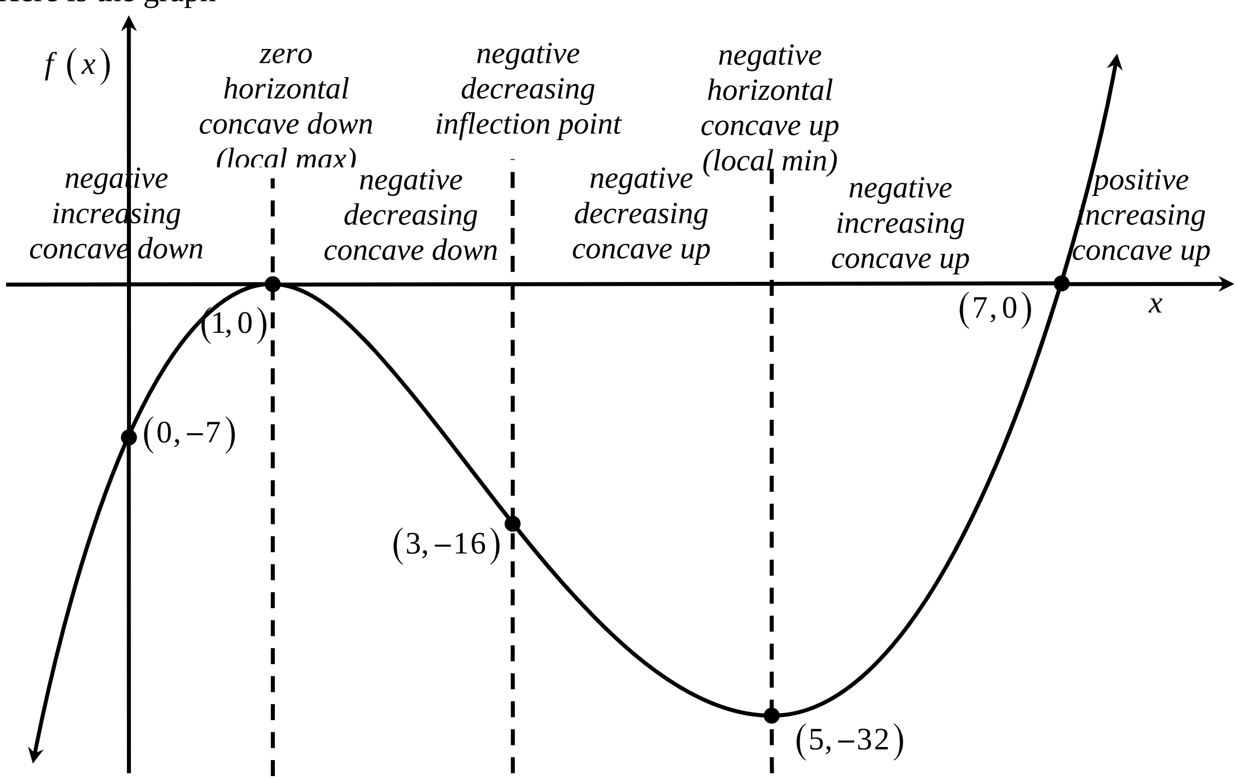
Step 8 & 9: $f''(x)=6x-18=6(x-3)$. The sign chart for f '' is:



sign chart for $f''(x)=6x-18=6(x-3)$

The y-value at the inflection point is $f(3)=-16$. So the inflection point is at $(3,-16)$.

Step 10: Here is the graph



graph of $f(x)=x^3-9x^2+15x-7$