

**Math 163A Handout 8: Worksheet for the “D-test”**

Goal: Find the  $(x, y)$  pairs that cause saddle points or relative extrema in the graph of  $f(x, y) =$

Step 1: Find  $f_x(x, y)$ .

Step 2: Find  $f_y(x, y)$ .

Step 3: Find  $f_{xy}(x, y)$ .

Step 4: Find  $f_{xx}(x, y)$ .

Step 5: Find  $f_{yy}(x, y)$ .

Step 6: Build  $D(x, y)$ . Remember,  $D(x, y) = f_{xx}(x, y) \cdot f_{yy}(x, y) - (f_{xy}(x, y))^2$ .

continued on back



Step 7: Find the critical pairs. They are the pairs  $(x, y) = (a, b)$  that cause both  $f_x(a, b) = 0$  and  $f_y(a, b) = 0$ .

Step 8: For each critical pair  $(x, y) = (a, b)$ , compute  $D(a, b)$  and note whether it is positive, negative, zero.

Step 9: For each critical pair  $(x, y) = (a, b)$ , also compute  $f_{xx}(a, b)$ .

Step 10: Remember the following criteria about the behavior of critical pairs.

- If a critical pair  $(x, y) = (a, b)$  has  $D(a, b) = \textit{negative}$ , then you can immediately conclude that there will be a saddle point in the graph of  $f$  at  $(x, y) = (a, b)$ .
- If a critical pair  $(x, y) = (a, b)$  has  $D(a, b) = 0$ , then you can immediately conclude that you are out of luck: the “D-test” does not tell you what happens in the graph of  $f$  at  $(x, y) = (a, b)$ .
- If a critical pair  $(x, y) = (a, b)$  has  $D(a, b) = \textit{positive}$  and  $f_{xx}(a, b) = \textit{positive}$ , then there will be a relative min in the graph of  $f$  at  $(x, y) = (a, b)$ .
- If a critical pair  $(x, y) = (a, b)$  has  $D(a, b) = \textit{positive}$  and  $f_{xx}(a, b) = \textit{negative}$ , then there will be a relative max in the graph of  $f$  at  $(x, y) = (a, b)$ .

Using the criteria above, analyze each critical pair to decide if it is a saddle, max, min, etc. Write your conclusions clearly.