

Things to know before MATH 263A at Ohio University

Because *math is cumulative*, mastery and review of previous material is essential for your success in Calculus. Your instructor will take for granted that you have mastery of the material below. You should understand, be able to use, and memorize all of the material and formulas on these pages. You should review it before every test.

Algebra

How to factor a polynomial.

How to solve a system of linear equations.

How to complete the square.

Understand, *memorize* and be able to use all formulas:

Arithmetic operations:

$$a(b + c) = ab + ac$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Caution: $\frac{a + b}{c + d} \neq \frac{a}{c} + \frac{b}{d}$.

Exponents and radicals:

$$x^m x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^n = x^n y^n$$

$$x^{1/n} = \sqrt[n]{x}$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$x^{-n} = \frac{1}{x^n}$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Caution: $\sqrt{x^2 + y^2} \neq x + y$.

Factoring Special Polynomials:

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Binomial Theorem:

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Quadratic Formula:

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Inequalities and Absolute Values:

If $a < b$ and $b < c$, then $a < c$

If $a < b$, then $a + c < b + c$

If $a < b$ and $c > 0$ then $ac < bc$

If $a < b$ and $c < 0$ then $ac > bc$

If $a > 0$ then:

$|x| = a$ means $x = a$ or $x = -a$

$|x| < a$ means $-a < x < a$

$|x| > a$ means $x < -a$ or $x > a$

Geometry

Understand, *memorize* and be able to use all formulas:

Formulas for area A , circumference C , and volume V .

Triangle:

$$A = \frac{1}{2}bh = \sin \theta$$

Circle:

$$A = \pi r^2, C = 2\pi r$$

Sector of a circle:

$$A = \frac{1}{2}r^2\theta, s = r\theta \text{ (in radians)}$$

Sphere:

$$V = \frac{4}{3}\pi r^3, A = 4\pi r^2$$

Cylinder:

$$V = \pi r^2 h$$

Cone:

$$\frac{1}{3}\pi r^2 h$$

Distance and Midpoint Formulas:

Distance between points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Midpoint of $\overline{P_1P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Lines:

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept line equation: $y = mx + b$.

Point-slope line equation: $y - y_0 = m(x - x_0)$.

Circles: Equations of the circle with center (h, k) and radius r :

$$(x - h)^2 + (y - k)^2 = r^2$$

Conic sections. Know how to graph them:

Ellipse: $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1,$

Parabola: $4p(y - k) = (x - h)^2$

Hyperbola: $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1,$

Trigonometry

Understand, memorize, be able to use all formulas:

Angle Measurement:

$$\pi \text{ radians} = 180^\circ \quad s = r\theta$$

$$1^\circ = \frac{\pi}{180} \text{ rad} \quad 1 \text{ rad} = \frac{180^\circ}{\pi}$$

Trigonometric Functions:

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{r}{y}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y}$$

Fundamental Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

Caution: $\sin^2 \theta = (\sin \theta)^2 \neq \sin(\theta^2)$.

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Half-angle Formulas:

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

Memorize the graphs of $\sin x$, $\cos x$ and $\tan x$.

Trigonometric functions of important angles:

θ	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	---

Functions and Graphs

Review sections 1.1 and 1.2 of the text!

Know all formulas as well as functions and graphs:

Power Functions. Know the graphs of:

$$f(x) = x^n \text{ for any integer } n$$

$$f(x) = \sqrt[n]{x} = x^{1/n} \text{ for } n = 2, 3$$

$$f(x) = x^{-1} = \frac{1}{x}$$

$$f(x) = x^{-2} = \frac{1}{x^2}$$

Exponential and Logarithmic Functions. Know the graphs.:

$$\log_a x = y \iff a^y = x$$

$$\ln x = y \iff e^y = x$$

$$\ln x = \log_e x$$

e^x is sometimes written $\exp(x)$.

$$\ln e = 1$$

Cancellation Equations:

Laws of Logarithms

$$\log_a(a^x) = x \quad a^{\log_a x} = x$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\ln(e^x) = x \quad e^{\ln x} = x$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^r) = r \log_a x$$

Hyperbolic Functions. Know the definitions and graphs.:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{coth} x = \frac{\cosh x}{\sinh x}$$

How the difference between the Graph of an Equation and the Graph of a Function

Translation Principles. For the graph of any equation:

$x \mapsto (x - h)$ shifts right by h , $y \mapsto (y - k)$ shifts up by k .