Things to know before MATH 263B at Ohio University

Because *math is cumulative*, mastery and review of previous material is essential for your success in Calculus. Your instructor will take for granted that you have mastery of all the material below. You should understand, be able to use, and memorize all of the material and formulas on these pages. You should review it before every test.

You should begin by reviewing "Things to Know before MATH 263 A" on the class web site. In addition you should know the following:

Continuity: at a means: $\lim_{x \to a} f(x) = f(a)$.
All elementary functions: polynomial, rational, root, trig., exponential and logarithmic functions are continuous whereever defined.

Definition of derivative: $f'(x) \equiv \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

Differentiable at a point means the derivative exists there.
Theorem: If a function is differentiable at a, then it is continuous at a.
Differentiability can fail because: (a) corner, (b) discontinuity, (c) vertical tangent.
Higher derivatives: $f''(x)$.

Meaning of the Derivative:
$f'(a)$ is the slope, m, of the tangent line to $y = f(x)$ at $(a, f(a))$.
$f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$.
If $x(t)$ is position, $x'(t)$ is velocity and $x''(t)$ is acceleration.
Be able to estimate derivative from data or a graph.

Differentiation rules:
Memorize and be able to use all of these.

$$\frac{d}{dx}c = 0.$$

$$\frac{d}{dx}\big(af(x) + bg(x)\big) = af'(x) + bg'(x).$$

$$\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x).$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \text{ (Chain rule)}$$

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u \tan u\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\sinh u = \cosh u\frac{du}{dx}$$

$$\frac{d}{dx}\cosh u = \sinh u\frac{du}{dx}$$

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\tan^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx}$$

Find domains of the above functions.

Basic Trig. Limit: $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

Linear approximations
- $f(x) \approx L(x) \equiv f(x_0) + f'(x_0)(x - x_0)$.

Logarithmic differentiation - use ln( ) to simplify products, sums and exponents, then differentiate.

Implicit Differentiation:

Means: Differentiate the whole equation: $\frac{d}{dx}(\ldots = \ldots)$.
Assume y is a function of x, so when you differentiate y you get dy/dx. When finding dy/dx at a specific point you may plug in the point immediately.

Related Rates:

1. Draw a picture of the moving variables.
2. Write an equation that relates variables.
3. Implicitely differentiate the equation with respect to t.
4. Plug in values.

l'Hopital's Rule
$\lim_{x\to a} f(x)/g(x)$. f and g should be differentiable and g not zero near a. Use L'Hopital's rule for $\frac{0}{0}$ and $\frac{\infty}{\infty}$.
Change $\infty \cdot 0$ and $\infty - \infty$ to $\frac{0}{0}$ or $\frac{\infty}{\infty}$.
Use ln( ) for 0^0, ∞^0, 1^∞

Max-Min Questions:
- First Derivative Test.
  Check $f'(x)$ on each side of the critical point.
- Second Derivative Test
  Check $f''(x)$ at the critical point.
- On a finite, closed interval, must check for max./min. at critical points and endpionts.
- Memorize all area and volume formulas.
- On applied problems, draw a picture with labels. Be sure to correctly identify the variable that you want to maximize or minimize.

Mean Value Theorem
- Memorize the theorem exactly.
- Be able to decide whether or not it applies to a given function, giving reasons why it is continuous and differentiable (b/c it is a certain class of function, e.g. polynomial, trig., etc.).
- Be able to find c that satisfies the theorem. First calculate the mean change (this is a number) $m = (f(b) - f(a))/(b - a)$, then solve $f'(c) = m$ for c.

General
The Chain Rule is really important. Know how to use it properly.
On exams, explain anything you can.
If you get stuck on a problem on an exam, tell what you would do if you could.