Part V
Appendices

Appendix A

Sample Exams

MATH344 Midterm Exam – Spring 04

1. For $f(x) = x^2 - 5$, do 2 iterations of Newton's method, starting with $x_0 = 2.0$. What is the relative error of x_2? About how many more steps would be needed to make the error less than 10^{-16}?
2. Write a **Matlab** program to do n steps of the bisection method for a function f with starting interval $[a, b]$. Let f, a, b and n be the inputs and the final x the output.
3. Given a data set represented by vectors $\mathbf{x}$ and $\mathbf{y}$, describe how you would use **Matlab** to get a Least Squares Approximation, Polynomial Interpolation and Spline Interpolation?
4. Given that the LU decomposition of $A = \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$ is $LU = \begin{bmatrix} 1 & 0 \\ 1/3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 0 & 1 \end{bmatrix}$, solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = (1, 2)'$.
5. Suppose $A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$. Using $\mathbf{v}_0 = (1, 1)'$ as the starting vector do 2 iterations of the Inverse Power Method for A. What do the results mean?
6. Let $A = \begin{bmatrix} 1 & .5 \\ 2 & 1 \end{bmatrix}$. Find the LU factorization with pivoting.
7. What is the condition number of a matrix? How do you find it with **Matlab**? What are the implications of the condition number when solving a linear system? What is the engineering solution to a problem with a bad condition number?
8. Write a **Matlab** program to do n iterations of the Power Method. Let the matrix A and n be inputs and let [e v] (the eigenvalue and eigenvector) be the outputs.
9. Write a **Matlab** program to that solves a linear system $A\mathbf{x} = \mathbf{b}$ using LU decomposition. Let A, $\mathbf{b}$ and *tol* be the inputs and $\mathbf{x}$ the output. If the error (residual) is not less than *tol*, then display a warning.
10. List your 10 least favorite **Matlab** commands.

X. What problem did Dr. Young totally botch in class?

MATH 344 – Midterm Exam – Winter 05

1. For $f(x) = x^2 - 5$, do 2 iterations of the bisection method, starting with $[a, b,] = [2, 3]$. What is the relative error? About how many more steps would be needed to make the error less than 10^{-6}?
2. Write a MATLAB program to do n steps of Newton's method for a function f with starting interval $[a, b]$. Let f, f', x_0 and n be the inputs and the final x the output.
3. Suppose $f(x)$ has been defined as an inline function. Give MATLAB commands to plot it on the interval $[0, 10]$.
4. Write a function program which will find the roots of a function f on an interval $[a, b]$.
5. Suppose $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$. Using $\mathbf{v}_0 = (1, 1)'$ as the starting vector do 2 iterations of the Power Method for A. What do the results mean?
6. Let $A = \begin{bmatrix} -1 & 5 \\ 2 & 2 \end{bmatrix}$. Find the LU factorization with pivoting.
7. What is the condition number of a matrix? How do you find it with MATLAB? What are the implications of the condition number when solving a linear system? What is the engineering solution to a problem with a bad condition number?
8. Write a MATLAB program to do n iterations of the QR Method. Let the matrix `A` and `n` be inputs and let `e` be the output.
9. Write a MATLAB program to that solves a linear system $A\mathbf{x} = \mathbf{b}$ using LU decomposition. Let A, $\mathbf{b}$ and *tol* be the inputs and $\mathbf{x}$ the output. If the error (residual) is not less than *tol*, then display a warning.
10. Give the MATLAB commands, or sequences of commands for solving a linear system $A\mathbf{x} = \mathbf{b}$ in as many ways as you know. Which of these are the worst and best?

X. When using Newton's method, how does one measure its effectiveness?

MATH 344 – Midterm Exam – Spring 2006

1. What 10 commands in MATLAB are the least useful?

2. For $f(x) = x^3 - 6$, do 2 iterations of Newton's method, starting with $x_0 = 2$.

3. What are the main *differences* in the uses of: Polynomial Interpolation, Splines and Least Squares Fitting?

4. Find the LU decomposition of A using pivoting if needed:
   $A = \begin{bmatrix} 3 & -2 \\ 6 & 1 \end{bmatrix}$

5. Write a MATLAB function program that calculates the sum of the squares of the first n integers.

6. What is the command in MATLAB to produce the eigenvalues and eigenvectors of a matrix. Which method does it use? What will be the form of the output?

7. What is the condition number of a matrix? How do you find it with MATLAB? What are the implications of the condition number when solving a linear system?

8. Find the eigenvalues and eigenvectors of the matrix:
   $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

9. Write a MATLAB function program that takes an input n, produces a random $n \times n$ matrix A and random vector $\bar{b}$, solves $A\bar{x} = \bar{b}$ (using the built in command) and outputs the residual (number).

10. Write a MATLAB script program that will use Newton's method to find a root of the system of functions $f_1(x, y) = x^3 - y^2 + 1$ and $f_2(x, y) = y^3 + x - 1$ starting from the initial guess $(0, 0)$.

X. In this class, every problem leads to ... .

MATH344 Final Exam – Spring 2004

1. Estimate the integral $\int_0^\pi \sin x\,dx$ using L_4, R_4 and T_4. Calculate the exact value and the percentage errors of each of the approximations.
2. Write a MATLAB program to do the midpoint method for integration. Let the inputs be the function f, the endpoints a, b and the number of subintervals n.
3. Write a MATLAB program to do n steps of the Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, with $\mathbf{x}(a) = \mathbf{x}_0$. Let the first line be:
`function [t, x] = myeuler(f,x0,a,b,n)`.
4. What is the condition number of a matrix? How do you find it with **MatLab**? What are the implications of the condition number when solving a linear system? What is the engineering solution to a problem with a bad condition number?.
5. Write the equation $\theta" + a\theta' + b\sin\theta = c\sin t$ as a linear system of first order equations and set up the Euler method for the system.
6. Describe RK45. What is the command for it in MATLAB?
7. When and why does the explicit finite difference method for the heat/diffusion equation become unstable?
8. Derive the implicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$.
9. Set up the finite difference equations for the BVP: $u_{xx} + u_{yy} = f(x, y)$, on the rectangle $0 \le x \le a$ and $0 \le y \le b$, with $u = 0$ on all the boundaries. Explain how the difference equations could be solved as a linear system.
10. Write a MATLAB program to do Newton's method with starting point x_0 that does n steps or stops when a tolerance is reached. Include f' in the inputs. Let the first line be: `function x = mynewton(f,f1,x0,n,tol)`.
11. What is the geometric meaning of eigenvalue and eigenvector. What is the MATLAB command to find them, with correct syntax. Describe an application of ew's and ev's.
12. Give the MATLAB command(s) for as many different ways as you know to solve a system of linear equations. Rank them.
13. Discuss uses of Polynomial Interpolation, Splines and Least Squares Interpolations.
14. What is a finite element and a finite element solution?
15. Very generally, how are the boundary and interior values of the finite element solution obtained.

X. When you have two different methods of approximating something, how can you get an even better approximation?

MATH344 – Final Exam – Winter 2005

1. Estimate the integral $\int_0^{16} \sqrt{x}\,dx$ using L_4, R_4, T_4 and S_4. Calculate the exact value and the percentage errors of each of the approximations.
2. Write a MATLAB function program to do the trapezoid method for integration. Let the inputs be the function f, the endpoints a, b and the number of subintervals n.
3. Write a MATLAB program to do n steps of the modified Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, with $\mathbf{x}(a) = \mathbf{x}_0$. Let the first line be:
`function [t, x] = myeuler(f,x0,a,b,n)`.
4. Write a MATLAB script program that calculates $\displaystyle\sum_{i=0}^{\infty} \frac{.5^i}{i^2} \sin(i)$ by adding terms until the sum stops changing. How do you know that it would stop?
5. Write the IVP: $\theta" + a\theta' + b\sin\theta = c\sin t$, $\theta(0) = \pi/4$, $\theta'(0) = 0$ as a system of first order equations. Give all the MATLAB commands needed to solve this IVP.
6. Describe RK45. What is the command for it in MATLAB?
7. Explain why order matters in engineering problems.
8. Derive the explicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$, with boundary conditions, $u(0,t) = a$, $u(L,t) = b$, and $u(x,0) = f(x)$.
9. Set up the finite difference equations for the BVP: $u_{rr} + \frac{1}{r}u_r = f(r)$, on the interal $0 \le r \le R$, with $u(R) = 0$ and $u_r(0) = 0$. Explain how to avoid the problem at $r = 0$.
10. Write a MATLAB program to do Newton's method with starting point x_0 that does n steps or stops when a tolerance is reached. Include f' in the inputs. Let the first line be:
`function x = mynewton(f,f1,x0,n,tol)`.
11. Write a MATLAB program to do n iterations of the inverse power method. Let the matrix A and n be the inputs and `[e v]` be the output. What is the meaning of e and v?
12. Describe methods for approximating double integrals.
13. What are: Polynomial Interpolations, Splines and Least Squares Interpolations. How do you get them from MATLAB?
14. Discuss: triangulation, finite element and finite element solution.
15. How are the boundary and interior values of the finite element solution obtained.

X. When I can't get my MATLAB program to work, I ... .

MATH344 – Final Exam – Spring 2006

1. Approximate the integral $\int_0^\pi \sin x\,dx$ using M_4 and S_4. Which do you expect to be more accurate?
2. Write a MATLAB function program to do the Trapezoid Rule for integration of data. Let the inputs be vectors x and y, where it is assumed that y is a function of x and x is not necessarily evenly spaced.
3. Describe and give formulas for 2 methods to approximate double integrals based on triangles.
4. Explain how to incorporate an insulated boundary in a finite difference method.
5. Set up the finite difference equations for the BVP: $u_{rr}+\frac{1}{r}u_r = f(r)$, on the interval $0 \le r \le R$, with $u(R) = 4$ and $u_r(0) = 0$. Explain how to avoid the problem at $r = 0$.
6. Do 2 steps of Newton's method to solve the equation $f(x) = x^2 - 5 = 0$, with starting point $x_0 = 2$. Find the percentage errors of x_0, x_1 and x_2.
7. Write a MATLAB script program that will use Newton's method to find a root of the system of functions $f_1(x, y) = x^3 + y - 1$ and $f_2(x, y) = y^3 - x + 1$ starting from the initial guess $(.5, .5)$.
8. Write a MATLAB function program to do n steps of the Modified Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x})$, on the time interval $[a, b]$, with $\mathbf{x}(a) = \mathbf{x}_0$.
9. Write the IVP: $\theta'' + .5\theta' + \sin\theta = \sin 2t$, $\theta(0) = 1$, $\theta'(0) = 0$ as a system of first order equations. Give all the MATLAB commands needed to solve this IVP on the interval $0 \le t \le 10$.
10. What is variable step size? How is it implemented RK45?
11. What are main differences between the Finite Difference Method and Finite Elements Method?
12. If $U(x) = \sum_{j=1}^{n} C_j \Phi_j(\bar{x})$ is a finite element solution, what is the meaning of C_j? What is the final step of finding a finite element solution?
13. Write a MATLAB program to do n iterations of the power method. Let the matrix A and n be the inputs and `[e v]` be the output. What is the meaning of e and v?
14. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Find the LU factorization with pivoting.
15. What is a polynomial interpolation? How do you get one in MATLAB?

X. When you have two different methods to make an approximation, how can you get a potentially more accurate method?

Appendix B

Glossary of Matlab Commands

Mathematical Operations

`+` Addition. Type `help plus` for information.
`-` Subtraction. Type `help minus` for information.
`*` Scalar or matrix multiplication. Type `help mtimes` for information.
`/` Scalar or right matrix division. Type `help slash` for information.
For matrices, the command `A/B` is equivalent to `A*inv(B)`.
`^` Scalar or matrix powers. Type `help mpower` for information.
`.*` Element by element multiplication. Type `help times` for information.
`.^` Element by element exponentiation. Type `help power` for information.
`./` Element by element division.

Built-in Mathematical Constants

`eps` Machine epsilon, i.e. approximately the computer's floating point roundoff error.
`i` $\sqrt{-1}$.
`Inf` ∞.
`NaN` Not a number. Indicates an invalid operation such as $0/0$.
`pi` $\pi = 3.14159\ldots$.

Built-in Mathematical Functions

`abs(x)` Absolute value$|x|$.
`acos(x)` Inverse cosine $\arccos x$.
`asin(x)` Inverse sine $\arcsin x$.

`atan(x)` Inverse tangent $\arctan x$.
`cos(x)` Cosine $\cos x$.
`cosh(x)` Hyperbolic cosine $\cosh x$.
`cot(x)` Cotangent $\cot x$.
`exp(x)` Exponential function $e^x = \exp x$.
`log(x)` Natural logarithm $\ln x = \log_e x$.
`sec(x)` Secant $\sec x$.
`sin(x)` Sine $\sin x$.
`sinh(x)` Hyperbolic sine $\sinh x$.
`sqrt(x)` Square root $\sqrt{x}$.
`tan(x)` Tangent $\tan x$.
`tanh(x)` Hyperbolic tangent $\tanh x$.
`max` Computes maximum of the rows of a matrix.
`mean` Computes the average of the rows of a matrix.
`min` Computes the minimum of the rows of a matrix.

Built-in Numerical Mathematical Operations

`fzero` Tries to find a zero of the specified function near a starting point or on a specified interval.
`inline` Define a function in the command window.
`ode113` Numerical multiple step ODE solver.
`ode45` Runga-Kutta 45 numerical ODE solver.
`quad` Numerical integration using an adaptive Simpson's rule.
`dblquad` Double integration.
`triplequad` Triple integration.

Built-in Symbolic Mathematical Operations

`collect` Collects powers of the specified variable is a given symbolic expression.
`compose` Composition of symbolic functions.
`diff` Symbolic differentiation.
`double` Displays double-precision representation of a symbolic expression.
`dsolve` Symbolic ODE solver.
`expand` Expands an algebraic expression.
`factor` Factor a polynomial.
`int` Symbolic integration; either definite or indefinite.

`limit` Finds two-sided limit, if it exists.
`pretty` Displays a symbolic expression in a nice format.
`simple` Simplifies a symbolic expression.
`subs` Substitutes for parts a a symbolic expression.
`sym` or `syms` Create symbolic variables.
`symsum` Performs a symbolic summation, possibly with infinitely many entries.
`taylor` Gives a Taylor polynomial approximation of a given order at a specified point.

Graphics Commands

`contour` Plots level curves of a function of two variables.
`contourf` Filled contour plot.
`ezcontour` Easy contour plot.
`loglog` Creates a log-log plot.
`mesh` Draws a mesh surface.
`meshgrid` Creates arrays that can be used as inputs in graphics commands such as `contour`, `mesh`, `quiver`, and `surf`.
`ezmesh` Easy mesh surface plot.
`plot` Plots data vectors.
`ezplot` Easy plot for symbolic functions.
`plot3` Plots curves in 3-D.
`polar` Plots in polar coordinates.
`quiver` Plots a vector field.
`semilogy` Semilog plot, with logarithmic scale along the vertical direction.
`surf` Solid surface plot.
`trimesh` Plot based on a triangulation `trisurf` Surface plot based on a triangulation

Special Matlab Commands

`:` Range operator, used for defining vectors and in loops. Type `help colon` for information.
`;` Suppresses output. Also separates rows of a matrix.
`=` Assigns the variable on the left hand side the value of the right hand side.
`ans` The value of the most recent unassigned.
`cd` Change directory.
`clear` Clears all values and definitions of variables and functions. You may also use to

clear only specified variables.

`diary` Writes a transcript of a MATLAB session to a file.

`dir` Lists the contents in the current working directory. Same as `ls`.

`help`

`inline` Define an inline function.

`format` Specifies output format, e.g. `> format long`.

`load` Load variables from a file.

`save` Saves workspace variables to a file.

Matlab Programming

`==` Is equal?

`~=` Is not equal?

`<` Less than?

`>` Greater than?

`<=` Less than or equal?

`break` Breaks out of a `for` or `while` loop.

`end` Terminates an `if`, `for` or `while` statement.

`else` Alternative in an `if` statement.

`error` Displays and error message and ends execution of a program.

`for` Repeats a block of commands a specified number of times.

`function` First word in a function program.

`if` Checks a condition before executing a block of statements.

`return` Terminates execution of a program.

`warning` Displays a warning message.

`while` Repeats a block of commands as long as a condition is true.

Commands for Matrices and Linear Algebra

Matrix arithmetic:

```
> A = [ 1  3 -2 5 ;  -1  -1 5 4 ; 0 1 -9  0] .....Manually enter a matrix.
> u = [ 1  2  3  4]'
> A*u
> B = [3 2 1; 7 6 5; 4 3 2]
> B*A ..... multiply B times A.
```

`> 2*A` ..... multiply a matrix by a scalar.
`> A + A` ..... add matrices.
`> A + 3` ..... add a number to every entry of a matrix.
`> B.*B` ..... component-wise multiplication.
`> B.^3` ..... component-wise exponentiation.

Special matrices:

`> I = eye(3)` ..... identity matrix
`> D = ones(5,5)`
`> O = zeros(10,10)`
`> C = rand(5,5)` ..... random matrix with uniform distribution in $[0, 1]$.
`> C = randn(5,5)` ..... random matrix with normal distribution.
`> hilb(6)`
`> pascal(5)`

General matrix commands:

`> size(C)` ..... gives the dimensions $(m \times n)$ of A.
`> norm(C)` ..... gives the norm of the matrix.
`> det(C)` ..... the determinant of the matrix.
`> max(C)` ..... the maximum of each row.
`> min(C)` ..... the minimum in each row.
`> sum(C)` ..... sums each row.
`> mean(C)` ..... the average of each row.
`> diag(C)` ..... just the diagonal elements.
`> inv(C)` ..... inverse of the matrix.

Matrix decompositions:

`> [L U P] = lu(C)`
`> [Q R] = qr(C)`
`> [U S V] = svm(C)` ..... singular value decomposition.

Bibliography

[1] Ayyup and McCuen, 1996.

[2] E. Johnston, J. Mathews, *Calculus*, Addison-Wesley, 2001