

# Lecture 23

## Plotting Functions of Two Variables

### Functions on Rectangular Grids

Suppose you wish to plot a function  $f(x, y)$  on the rectangle  $a \leq x \leq b$  and  $c \leq y \leq d$ . The graph of a function of two variables is of course a three dimensional object. Visualizing the graph is often very useful.

For example, suppose you have a formula

$$f(x, y) = x \sin(xy)$$

and you are interested in the function on the region  $0 \leq x \leq 5$ ,  $\pi \leq y \leq 2\pi$ . A way to plot this function in MATLAB would be the following sequence of commands:

```
f = @(x,y) x.*sin(x.*y)
[X,Y] = meshgrid(0:.1:5, pi:.01*pi:2*pi);
Z = f(X,Y)
mesh(X,Y,Z)
```

This will produce a 3-D plot that you can rotate by clicking on the rotate icon and then dragging with the mouse. Instead of the command `mesh`, you could use the command

```
surf(X,Y,Z)
```

The key command in this sequence is `[X Y] = meshgrid(a:h:b,c:k:d)`, which produces *matrices of x and y values* in X and Y. Enter:

```
size(X)
size(Y)
size(Z)
```

to see that each of these variables is a  $101 \times 51$  matrix. To see the first few entries of X enter

```
X(1:6, 1:6)
```

and to see the first few values of Y type

```
Y(1:6, 1:6)
```

You should observe that the  $x$  values in X begin at 0 on the left column and increase from left to right. The  $y$  values on the other have start at  $\pi$  at the top and increase from top to bottom. Note that this arrangement is flipped from the usual arrangement in the  $x$ - $y$  plane.

In the command `[X Y] = meshgrid(a:h:b,c:k:d)`,  $h$  is the increment in the  $x$  direction and  $k$  is the increment in the  $y$  direction. Often we will calculate

$$h = \frac{b-a}{m} \quad \text{and} \quad k = \frac{d-c}{n},$$

where  $m$  is the number of *intervals* in the  $x$  direction and  $n$  is the number of intervals in the  $y$  direction. To obtain a good plot it is best if  $m$  and  $n$  can be set between 10 and 100.

For another example of how `meshgrid` works, try the following and look at the output in `X` and `Y`.

```
[X,Y] = meshgrid(0:.5:4,1:.2:2);
```

## Scattered Data and Triangulation

Often we are interested in objects whose bases are not rectangular. For instance, data does not usually come arranged in a nice rectangular grid; rather, measurements are taken where convenient.

In MATLAB we can produce triangles for a region by recording the coordinates of the vertices and recording which vertices belong to each triangle. The following script program produces such a set of triangles:

```
% mytriangles
% Program to produce a triangulation.
% V contains vertices, which are (x,y) pairs
V = [ 1/2 1/2 ; 1 1 ; 3/2 1/2 ; .5 1.5 ; 0 0
      1 0 ; 2 0 ; 2 1 ; 1.5 1.5 ; 1 2
      0 2 ; 0 1]
% x, y are row vectors containing coordinates of vertices
x = V(:,1)';
y = V(:,2)';
% Assign the triangles
T = delaunay(x,y)
```

You can plot the triangles using

```
trimesh(T,x,y)
```

You can also prescribe values (heights) at each vertex directly (say from a survey):

```
z1 = [ 2 3 2.5 2 1 1 .5 1.5 1.6 1.7 .9 .5 ];
```

or using a function:

```
f = @(x,y) abs(sin(x.*y)).^(3/2);
z2 = f(x,y);
```

The resulting profiles can be plotted:

```
trimesh(T,x,y,z1)
trisurf(T,x,y,z2)
```

Each row of the matrix  $T$  corresponds to a triangle, so  $T(i, :)$  gives triangle number  $i$ . The three corners of triangle number  $i$  are at indices  $T(i, 1)$ ,  $T(i, 2)$ , and  $T(i, 3)$ . So for example to get the  $y$ -coordinate of the second point of triangle number 5, enter

```
| y(T(5,2))
```

To see other examples of regions defined by triangle get `mywedge.m` and `mywasher.m` from the web site and run them. Each of these programs defines vectors  $x$  and  $y$  of  $x$  and  $y$  values of vertices and a matrix  $T$ . As before  $T$  is a list of sets of three integers. Each triple of integers indicates which vertices to connect in a triangle.

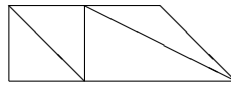
To plot a function, say  $f(x, y) = x^2 - y^2$  on the washer figure try

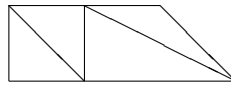
```
| mywasher
z = x.^2 - y.^2
trisurf(T,x,y,z)
```

Note again that this plot can be rotated using the icon and mouse.

## Exercises

- 23.1 Plot the function  $f(x, y) = xe^{-x^2-y^2}$  on the rectangle  $-3 \leq x \leq 3$ ,  $-2 \leq y \leq 2$  using `meshgrid` and `mesh`. Make an appropriate choice of  $m$  and  $n$  and if necessary a rotation to produce a good plot. Calculate the  $h$  and  $k$  corresponding to your  $m$  and  $n$ . Turn in your plot and the calculation of  $h$  and  $k$ .



- 23.2 Modeling after `mywasher.m`, produce  Use the `axis` command to zoom out so the outside edges are clearly visible.