1. Type \texttt{help rand} and read the first paragraph of resulting help page. Next enter the following:
\begin{verbatim}
x = rand(10,1)
y = x.^3
avg = sum(y)/10
\end{verbatim}
Figure out what happens in these command before you proceed.

2. Enter the following sequence commands:
\begin{enumerate}
\item[(a)] \texttt{n = 10}
\item[(b)] \texttt{x = rand(n,1); avg = sum(x.^3)/n}
\item[(c)] Use the ↑ key to recall this line and then press \texttt{Enter} again.
\item[(d)] Obtain 10 estimates this way and record the values you get along with the absolute error of each estimate. You can have \texttt{MatLAB} calculate the absolute error for you conveniently by including at the end of line of #1(b): \texttt{error = abs(.25 - avg)}.
\item[(e)] Explain why this is an approximation of $\int_0^1 x^3 \, dx$. (It has to do with the average of a function.)
\end{enumerate}

3. Enter the command \texttt{n = 100} and use the ↑ key to recall the line in #1(b) again. Press the enter key to execute this line. Obtain and record 10 estimates this way along with the absolute errors.

4. Repeat this process using \texttt{n = 1000} and \texttt{n = 10000}.

5. Make a chart showing the relationship between the sample size $n$ and the arithmetic mean of the absolute errors of the estimates with sample size $n$. The data should reflect the relationship $|E_n| \approx Kn^{-r}$. Use the data and logarithms to determine the constants $K$ and $r$.

6. For the Trapezoid rule $r = 2$ and for Simpson’s rule $r = 4$. How does the random method introduced here compare with the Trapezoid and Simpson’s methods of numerical integration? Which is the most accurate, which the least?

7. Prepare a brief written report answering all the questions. Use complete sentences and standard mathematical notation. Do not get a printout.

This demonstrates the connection between averages and integrals. Because this technique is efficient in higher dimensions, variants (known as Monte Carlo methods) are actually used in practice to evaluate integrals.

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