

Characterizations of some classes of rings via Rickart modules

ABSTRACT

Let M be a right R -module and $S = \text{End}_R(M)$. M is called a Rickart module if the right annihilator in M of any single element of S is generated by an idempotent in S .

In this talk, we will discuss characterizations of classes of rings using the Rickart property of modules over them. In particular, the classes of right semihereditary rings, right hereditary rings, von Neumann regular rings, V -rings and semisimple artinian rings, will be characterized. We will show that a commutative domain R is a Prüfer domain iff the free module $R^{(2)}$ is a Rickart R -module iff the free module $R^{(3)}$ has the SIP (Summand Intersection Property). We show that the SIP property for $R^{(2)}$ in the preceding result is not enough to get that R is Prüfer. We exhibit an example of a domain R for which the free R -module $R^{(2)}$ has the SIP, yet R is not a Prüfer domain. As an application of our results, we provide an alternate proof of an earlier result of Small by using the theory of Rickart modules: For any $k \in \mathbb{N}$, R is a right (semi)hereditary ring iff $M_k(R)$ is a right (semi)hereditary ring.

(This is a joint work with S. Tariq Rizvi and Cosmin Roman.)