Prime ideals in quantum algebras:
a short course by Tom Lenagan

In noncommutative algebraic geometry, the familiar points, curves, surfaces, etc. of ordinary geometry are replaced by the prime and primitive ideals and representations (modules) of "noncommutative algebras of functions", for example, of quantum groups and quantum algebras . Over the last few years, with several collaborators, I have been studying the prime ideal structure of various quantum algebras, especially quantum matrices and quantum grassmannians.

This course will introduce the main techniques employed in this study, with special reference to the methods introduced by Cauchon and Goodearl and Letzter to study quantum matrices and to the methods introduced by Launois, Rigal and myself to study the quantum grassmannian.

Many quantum algebras are acted on by a torus of automorphisms. The results of Goodearl and Letzter show that the study of the prime ideals left invariant by these automorphisms holds the clue to knowledge of all prime ideals. Cauchon's work enables one to begin to describe the these invariant prime ideals

In recent work with Launois and Rigal the invariant prime spectrum of the quantum grassmannian has been successfully studied via the notions of quantum algebras with a straightening law and the ideas of noncommutative Schubert cells and Schubert varieties. Remarkably, the description obtained is similar to the description of two different phenomena that have been studied recently. The first is the cell decomposition of the non-negative grassmannian, due to Postnikov, and the second is the study of torus orbits of symplectic leaves of Poisson algebras of matrices and grassmannians, due to Brown, Goodearl and Yakimov. These connections will be investigated and explanations given for the similarities.

At all stages the main results will be illustrated by reference to special examples, mainly using 2×2 quantum matrices and the 2×4 quantum grassmannian.