Divisible Abelian Groups Realizable as the Multiplicative Group of a Field

ABSTRACT

L. Fuchs posed the question of which abelian groups can be realized as the multiplicative group of nonzero elements of a field. Many partial results have been obtained (though it looks like a "nice" classification of such groups is untenable). In particular, the structure of the multiplicative group of algebraically closed and real closed fields was determined some time ago. More recently, Mott, Contessa, and Nichols proved that an infinite direct sum of copies of the rationals is isomorphic to the multiplicative group of some field. In this talk, we consider the more general class of divisible abelian groups and completely determine which groups in this class can be so realized. I plan to outline a completely elementary proof of Mott's result (the only algebra used is, essentially, just Lagrange's Theorem) using the compactness theorem of first order logic (you don't need to know much logic to follow this). Then I will discuss the classification of arbitrary "realizable" abelian groups. A few corollaries and applications will be included. To mention but one: if F is a field of characteristic 0 whose multiplicative group F^* is divisible, then F^* is isomorphic to K^*, where K is an algebraic closure of F, though F itself need not be algebraically closed. We contrast this with the characteristic p case.