Math 115 Section 03 (Barsamian) Midterm 1 Name (print):________________________

Ohio University, Friday 8 October, 2004

Attendance: $\frac{\quad}{24}$ Quizzes: $\frac{\quad}{100}$ Exams: $\frac{\quad}{200}$ Course: $\frac{\quad}{324}$ = % Course Grade:

Problem	1	2	3	4	5	6	7	8	9	10	11	**Total**
Your score												
Possible	20	15	20	20	15	15	20	15	20	25	15	**200**

Calculators are not allowed on this exam.

1 (20 points) Find all values of x that satisfy the inequality $\frac{x(x+2)}{(x-3)} \le 0$. Present your answer three ways: in a picture (using a number line), in interval notation, and in set notation. (Show your work.)

2 (15 points) Decide whether each function below has y-axis symmetry, origin symmetry, or neither.

(a) $y = 2 - 3x^4$ (Explain your answer.)

(b) $y = 2 - 3x^5$ (Explain your answer.)

(c) $y = 2x - 3x^5$ (Explain your answer.)

3 (20 points) If $f(x) = 1 + x^3$, find $f^{-1}(9)$. (Explain your answer.)

4 (20 points) Recall the definition of *average rate of change*:

Words: The *average rate of change* of the function f from $x = a$ to $x = b$

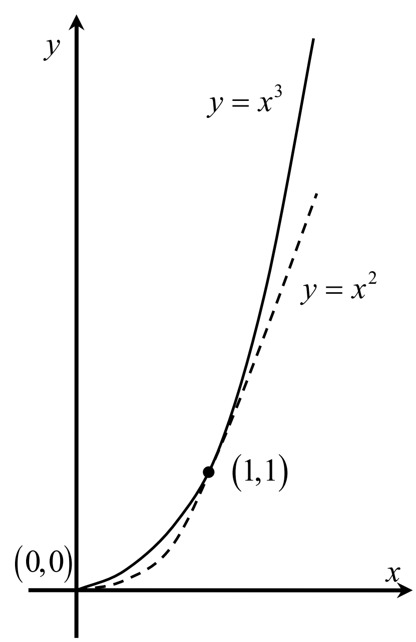
Meaning: $\dfrac{f(b) - f(a)}{b - a}$

Graphical interpretation: The number that is the slope of the "secant line" containing the points $(a, f(a))$ and $(b, f(b))$ on the graph of f.

(a) If $f(x) = \sqrt{x-3}$, find the average rate of change of f from $x = 4$ to $x = 7$.

(b) The number that you found in part (a) can be interpreted as the slope of a line segment on the graph of f. Sketch the graph of f and draw the line segment.

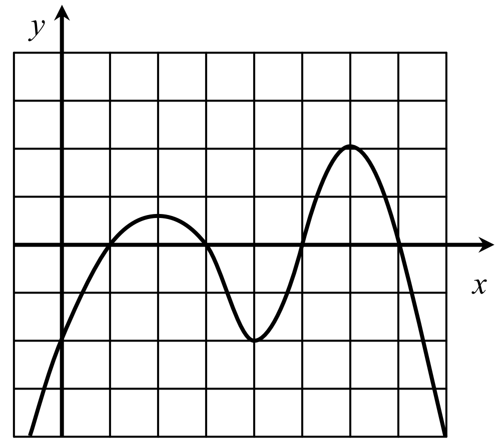
5 (15 points) A man in a trenchcoat tries to sell you this picture. He says that it shows the graph of the function $y = x^2$ as a dotted line, and the graph of the function $y = x^3$ as a solid line. You don't buy the picture, because you can see that there's something wrong with the graphs. Explain.



6 (15 points) Explain how you know that the graph to the right is NOT the graph of the function

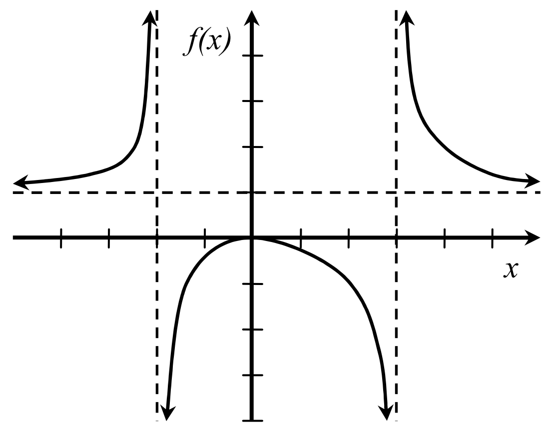
$$f(x) = x^3 - 9x^2 + 23x - 15 = (x-1)(x-3)(x-5).$$

There is more than one possible answer.



7 (20 points) Let $P(x) = 3x^4 + 2x^3 - x + 5$ and let $D(x) = x^2 + 2x - 1$. Using long division, find polynomials $Q(x)$ and $R(x)$ such that $P(x) = Q(x) \bullet D(x) + R(x)$. Show your work, and display your final results clearly.

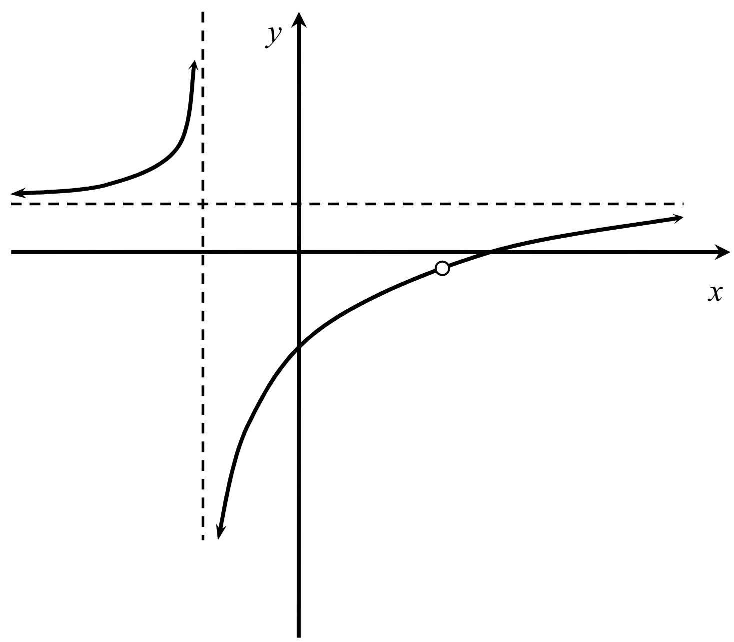
8 (15 points) (a) What is the domain of this function?



(b) What is the range?

9 (20 points) The graph of the rational function $f(x)=\frac{x^2-7x+12}{x^2-x-6}=\frac{(x-4)(x-3)}{(x+2)(x-3)}$ is shown.

Answer the following questions about the graph. In your answers, refer to the 7-step method for graphing rational functions, attached to the end of the exam.



(a) Why is there a hole in the graph?

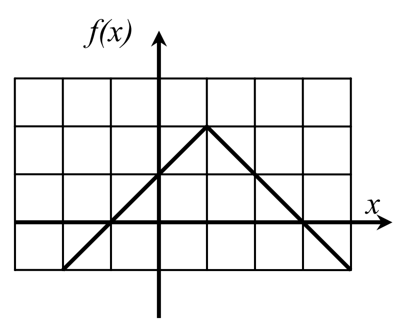
(b) Why is there a horizontal asymptote?

(c) What is the equation that describes the horizontal asymptote?

(d) What effect does the factor $(x-4)$ have on the graph?

10 (25 points) Sketch the graph of $f(x)=\dfrac{x-2}{x^2-2x-3}$. Use the 7-step method for graphing rational functions, attached to the end of the exam. (Show all your steps.)

11 (15 points) What function would have the graph shown to the right? (Explain your answer.)



<u>Seven Step Method for Graphing Rational Functions</u>

Math115 Precalculus (Barsamian)

Step 1: If $x = 0$ is in the domain, find $f(0)$.

Step 2: Check for symmetries.

Step 3: Determine the end behavior (horizontal asymptote? oblique asymptote? power function?) by deciding which of the following three cases applies.

case 1: degree of numerator < degree of denominator

In this case, the end behavior will resemble $y = \frac{1}{x^m}$. So there will be a horizontal asymptote at $y = 0$.

case 2: degree of numerator = degree of denominator

In this case, the end behavior will resemble $y = \frac{ax^m}{bx^m} = \frac{a}{b}x^0 = \frac{a}{b}$. So, a horizontal asymptote at $y = \frac{a}{b}$.

case 3: degree of numerator > degree of denominator

case 3a: degree of numerator = 1 + degree of denominator

In this case, the end behavior will resemble $y = mx + b$. So the line $y = mx + b$ will be an oblique asymptote. Perform long division of numerator by denominator to determine $mx + b$.

case 3b: degree of numerator $\geq$ 2 + degree of denominator

In this case, the end behavior will resemble $y = x^m$, for some integer $m \geq 2$.

Step 4: Factor the function (both numerator and denominator). This will enable you to determine the domain of the function, and it will also tell you the important x-values. (If a linear factor $(x - r)$ appears in the factorization, then the number r is an important x-value.)

Step 5: Make a sign chart. That is, put all the important x-values on a number line. In each region and at each important x-value, determine whether the function f is positive, negative, zero, or undefined.

Step 6: Locate vertical asymptotes, holes, and x-intercepts by examining the linear factors in the factorization. For each linear factor $(x - r)$ in the factorization, decide which of the following five cases applies.

case 1: The linear factor $(x - r)$ appears in the numerator but not in the denominator.

In this case, the graph will have an x-intercept at $x = r$.

case 2: The linear factor $(x - r)$ appears in the numerator and in the denominator but has a larger exponent in the numerator than it does in the denominator.

In this case, the graph will cross the x-axis at $x = r$, but there will be a hole at the crossing. (Technically, this is not called an "x-intercept", because $x = r$ is not in the domain.)

case 3: The linear factor $(x - r)$ appears in the numerator and in the denominator with equal exponents.

In this case, the graph will have a hole at $x = r$.

case 4: The linear factor $(x - r)$ appears in the numerator and in the denominator but has a smaller exponent in the numerator than it does in the denominator.

In this case, the graph will have a vertical asymptote at $x = r$.

case 5: The linear factor $(x - r)$ appears in the denominator only.

In this case, the graph will have a vertical asymptote at $x = r$.

Step 7: Based on the analysis in steps 1 through step 6, sketch the graph of f.