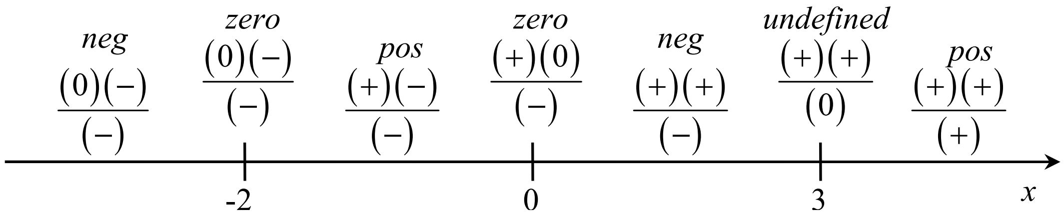
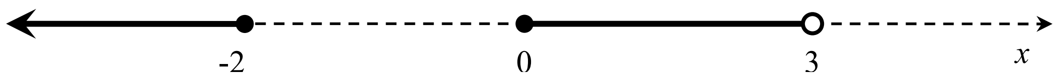
Math 115 Section 03 (Barsamian) Midterm 1 Solutions

1 Rewriting one of the factors and rearranging, $\frac{(x+2)(x-0)}{(x-3)} \le 0$, makes it clear that the important x-values are -2, 0, and 3. The sign chart becomes



The solution, presented as a picture, is



The solution, presented in interval notation, is $(-\infty,-2]\cup[0,3)$.

The solution, presented in set notation, is $\{x \mid x \le -2 \;\; OR \;\; 0 \le x < 3\}$.

2 (a) We build the three items $f(x)$, $f(-x)$, $-f(x)$, simplify them, and then compare them.

$$f(x) = 2-3x^4$$
$$f(-x) = 2-3x^4$$
These two match, so the function f has y-axis symmetry.
$$-f(x) = -2+3x^4$$

(b) We again build the three items $f(x)$, $f(-x)$, $-f(x)$, simplify them, and then compare them.

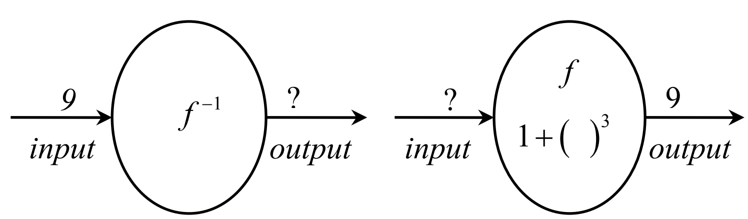
$$f(x) = 2-3x^5$$
$$f(-x) = 2+3x^5$$
$$-f(x) = -2+3x^5$$

Because none of the items match, the function has neither y-axis nor *origin* symmetry.

(c) We againbuild the three items $f(x)$, $f(-x)$, $-f(x)$, simplify them, and then compare them.

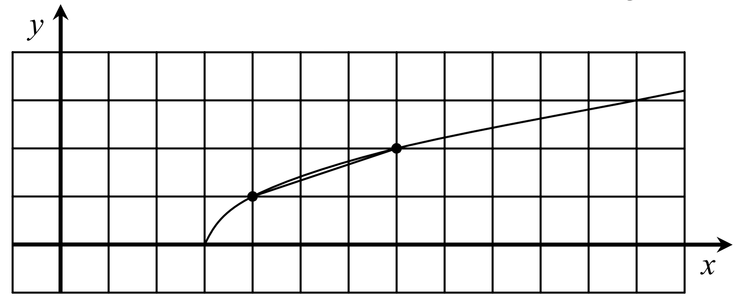
$$f(x) = 2x-3x^5$$
$$f(-x) = -2x+3x^5$$
$$-f(x) = -2x+3x^5$$
These two match, so the function f has *origin* symmetry.

3 The question is asking for the output of the function f^{-1} when the number 9 is used as *input*. This is the same as asking what number can be fed *into* the function f that will cause the number 9 to be *output*. We could visualize this in the machine diagrams shown at right. Therefore, we need to solve the following equation for x.



$$1+x^3 = 9$$
$$x^3 = 8$$
$$x = 2$$

4 $\frac{f(7)-f(4)}{7-4}=\frac{\sqrt{7-3}-\sqrt{4-3}}{7-4}=\frac{\sqrt{4}-\sqrt{1}}{3}=\frac{1}{3}$.



5 For $0<x<1$, the graph of $y=x^2$ should be above the graph of $y=x^3$. For example $\left(\frac{1}{2}\right)^2=\frac{1}{4}$, while $\left(\frac{1}{2}\right)^3=\frac{1}{8}$. This means that the graphs will have to cross at the point $(1,1)$.

6 Here are three good answers. There are others.

- The function has exactly three roots. So, the graph must have exactly three *x*-intercepts. It does not. The graph has four *x*-intercepts.
- The graph should have the end behavior of $y=x^3$. That means that it should go up on the right and down on the left. It doesn't. The graph goes down on both sides.
- The value of $f(0)$ is $f(0)=0^3-9(0)^2+23(0)-15=-15$. That means that the *y*-intercept of the graph should be at the point $(0,-15)$. It is not. The *y*-intercept of the graph is at the point $(0,-2)$.

7 Results presented in multiplication form:

$$P(x)=Q(x)\bullet D(x)+R(x)$$

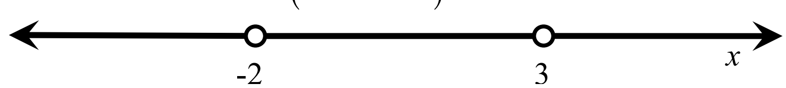
$$(3x^4+2x^3-x+5)=(3x^2-4x+11)\bullet(x^2+2x-1)+(-27x+16).$$

Results presented in division form:

$$\frac{P(x)}{D(x)}=Q(x)+\frac{R(x)}{D(x)}$$

$$\frac{(3x^4+2x^3-x+5)}{(x^2+2x-1)}=(3x^2-4x+11)+\frac{(-27x+16)}{(x^2+2x-1)}.$$

8 (a) the solution presented as a picture:

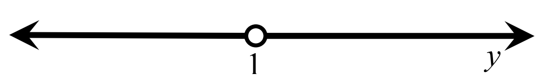


The solution presented in interval notation: $(-\infty,-1)\cup(-2,3)\cup(3,\infty)$.

The solution presented in set notation is: $\{x \mid x<-2 \;\; OR \;\; -2<x<3 \;\; OR \;\; 3<x\}$.

Alternate set notation is: $\{x \mid x\neq-2 \;\; AND \;\; x\neq3\}$.

(b) the solution presented as a picture:



The solution presented in interval notation: $(-\infty,1)\cup(1,\infty)$.

The solution presented in set notation is: $\{y \mid y<1 \;\; OR \;\; 1<y\}$.

Alternate set notation is: $\{y \mid y\neq1\}$.

9 (a) The linear factor $(x-3)$ appears in the numerator and denominator with equal exponents. Therefore, Step 6, case 3 applies. There will be a hole in the graph at the spot where $x=3$.

(b), (c) The degree of numerator = degree of denominator. Therefore, Step 3 case 2 applies. In this case, the end behavior will resemble $y=\frac{1x^m}{1x^m}=\frac{1}{1}x^0=\frac{1}{1}=1$. So, a horizontal asymptote at $y=1$.

(d) The linear factor $(x-4)$ appears in the numerator but not in the denominator. Therefore, Step 6 case 1 applies. That means there will be an x-intercept at *x*=4. (The *x*-intercept at *x*=4 would also show up on the sign chart in step 5.)

10 step 1: $f(0)=\frac{0-2}{0^2-2(0)-3}=\frac{-2}{-3}=\frac{2}{3}$. So the point $\left(0,\frac{2}{3}\right)$ will be on the graph.

step 2: To check for symmetry, we build the three items $f(x)$, $f(-x)$, and $-f(x)$, simplify them, and then compare them.

$$f(x)=\frac{x-2}{x^2-2x-3}$$

$$f(-x)=\frac{(-x)-2}{(-x)^2-2(-x)-3}=\frac{-x-2}{x^2+2x-3}$$

$$-f(x)=-\frac{x-2}{x^2-2x-3}=\frac{-(x-2)}{x^2-2x-3}=\frac{-x+2}{x^2-2x-3}$$

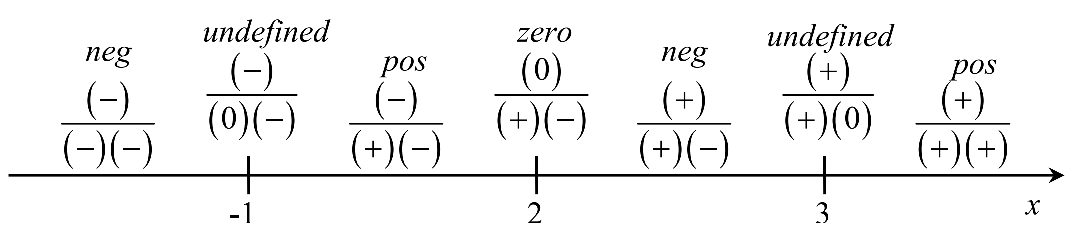
Because none of the items match, the graph of will have neither *y*-axis symmetry nor *origin* symmetry.

Step 3: The degree of the numerator is 1, while the degree of the denominator is 2. That means that degree of numerator < degree of denominator. So, Step 3 case 1 applies. In this case, the end behavior will resemble $y=\frac{1}{x}$. So there will be a horizontal asymptote at $y=0$.

Step 4: $f(x)=\frac{x-2}{x^2-2x-3}=\frac{(x-2)}{(x+1)(x-3)}$. Therefore, the domain of the function is: $\{x \mid x\neq -1 \ \ AND \ \ x\neq 3\}$.

The important *x* values are $x=-1,2,3$.

Step 5: The sign chart is



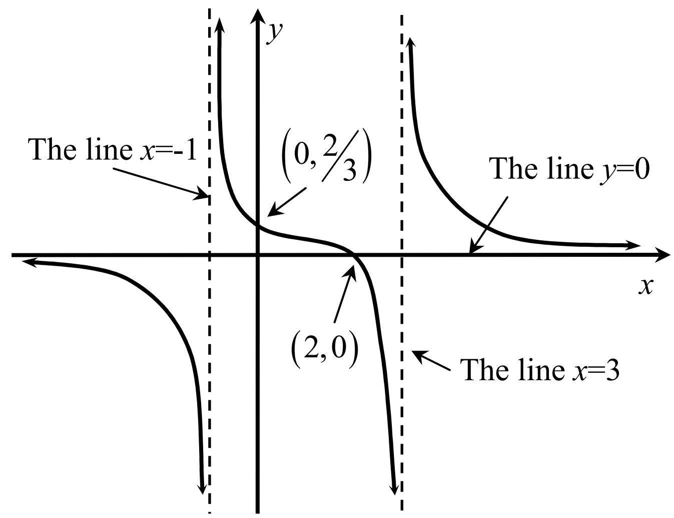
Step 6:

The linear factor $(x+1)$ appears in the denominator but not in the denominator. Therefore, Step 6 case 5 applies. That means there will be a vertical asymptote at *x*=-1.

The linear factor $(x-2)$ appears in the numerator but not in the denominator. Therefore, Step 6 case 1 applies. That means there will be an *x*-intercept at *x*=2.

The linear factor $(x-3)$ appears in the denominator but not in the denominator. Therefore, Step 6 case 5 applies. That means there will be a vertical asymptote at x=3.

Step 7:



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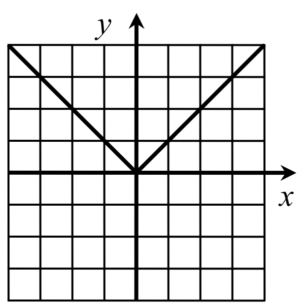
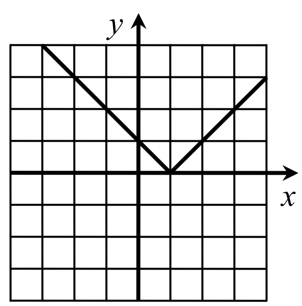
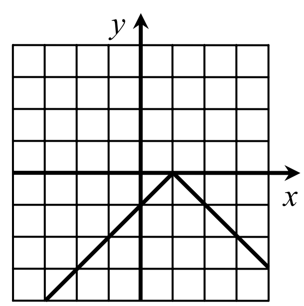


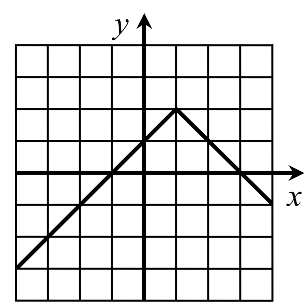
$y=|x|$



$y=|x-1|$



$y=-|x-1|$



$y=-|x-1|+2$