# Seven Step Method for Graphing Rational Functions

Math115 Precalculus (Barsamian)

***Step 1:*** If $x = 0$ is in the domain, find $f(0)$.

***Step 2:*** Check for symmetries.

***Step 3:*** Determine the end behavior (horizontal asymptote? oblique asymptote? power function?) by deciding which of the following three cases applies.

***case 1:*** degree of numerator < degree of denominator

In this case, the end behavior will resemble $y = \frac{1}{x^m}$. So there will be a horizontal asymptote at $y = 0$.

***case 2:*** degree of numerator = degree of denominator

In this case, the end behavior will resemble $y = \frac{ax^m}{bx^m} = \frac{a}{b}x^0 = \frac{a}{b}$. So, a horizontal asymptote at $y = \frac{a}{b}$.

***case 3:*** degree of numerator > degree of denominator

***case 3a:*** degree of numerator = 1 + degree of denominator

In this case, the end behavior will resemble $y = mx + b$. So the line $y = mx + b$ will be an oblique asymptote. Perform long division of numerator by denominator to determine $mx + b$.

***case 3b:*** degree of numerator $\geq$ 2 + degree of denominator

In this case, the end behavior will resemble $y = x^m$, for some integer $m \geq 2$.

***Step 4:*** Factor the function (both numerator and denominator). This will enable you to determine the domain of the function, and it will also tell you the important $x$-values. (If a linear factor $(x - r)$ appears in the factorization, then the number $r$ is an important $x$-value.)

***Step 5:*** Make a sign chart. That is, put all the important $x$-values on a number line. In each region and at each important $x$-value, determine whether the function $f$ is positive, negative, zero, or undefined.

***Step 6:*** Locate vertical asymptotes, holes, and $x$-intercepts by examining the linear factors in the factorization. For each linear factor $(x - r)$ in the factorization, decide which of the following five cases applies.

***case 1:*** The linear factor $(x - r)$ appears in the numerator but not in the denominator.

In this case, the graph will have an $x$-intercept at $x = r$.

***case 2:*** The linear factor $(x - r)$ appears in the numerator and in the denominator but has a larger exponent in the numerator than it does in the denominator.

In this case, the graph will cross the $x$-axis at $x = r$, but there will be a hole at the crossing. (Technically, this is not called an "$x$-intercept", because $x = r$ is not in the domain.)

***case 3:*** The linear factor $(x - r)$ appears in the numerator and in the denominator with equal exponents.

In this case, the graph will have a hole at $x = r$.

***case 4:*** The linear factor $(x - r)$ appears in the numerator and in the denominator but has a smaller exponent in the numerator than it does in the denominator.

In this case, the graph will have a vertical asymptote at $x = r$.

***case 5:*** The linear factor $(x - r)$ appears in the denominator only.

In this case, the graph will have a vertical asymptote at $x = r$.

***Step 7:*** Based on the analysis in steps 1 through step 6, sketch the graph of $f$.