Math 306 Section 02 (Barsamian) Induction

New Rule of Inference: The Principle of Induction

$P(a)$ is true
For all integers $k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true.
$\therefore$ For all integers $n \geq a$, $P(n)$ is true.

Usage:

- The letter "*a*" represents some fixed integer.
- The letters "*k*" and "*n*" represent variables whose domain *D* is the set of all integers greater than or equal to "*a*".
- The symbol $P(n)$ represents a predicate whose domain is the set *D*.

This new rule of inference will be used to prove statements of the form

"For all integers $n \geq a$, $P(n)$ is true."

Strategy for using the principle of induction

Preliminary work:

- Identify the number playing the role of "*a*". (Introduce it.)
- Identify the predicate $P(n)$. (Introduce it in a sentence.)
- Figure out what the expressions for $P(a)$, $P(k)$, and $P(k+1)$ look like. (Write them down.)

Build a proof using the following structure:

Proof that for all integers $n \geq a$, $P(n)$ is true:

Basis Step: Prove that $P(a)$ is true.

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* a bunch of steps may be involved
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Inductive Step: Prove that for all integers $k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true.

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* a bunch of steps may be involved
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Conclusion: Therefore, for all integers $n \geq a$, $P(n)$ is true. (by the principle of induction)

End of Proof