Seven Step Method for Graphing Rational Functions

Math 163A Calculus (Barsamian)

Step 1: If $x = 0$ is in the domain, find $f(0)$.

Step 2: Check for symmetries.

Step 3: Determine the end behavior (horizontal asymptote? oblique asymptote? power function?) by deciding which of the following three cases applies.

case 1: degree of numerator < degree of denominator

In this case, the end behavior will resemble $y = \frac{1}{x^m}$. So there will be a horizontal asymptote at $y = 0$.

case 2: degree of numerator = degree of denominator

In this case, the end behavior will resemble $y = \frac{ax^m}{bx^m} = \frac{a}{b}x^0 = \frac{a}{b}$. So, a horizontal asymptote at $y = \frac{a}{b}$.

case 3: degree of numerator > degree of denominator

case 3a: degree of numerator = 1 + degree of denominator

In this case, the end behavior will resemble $y = mx + b$. So the line $y = mx + b$ will be an oblique asymptote. Perform long division of numerator by denominator to determine $mx + b$.

case 3b: degree of numerator $\geq$ 2 + degree of denominator

In this case, the end behavior will resemble $y = x^m$, for some integer $m \geq 2$.

Step 4: Factor the function (both numerator and denominator). This will enable you to determine the domain of the function, and it will also tell you the important *x*-values. (If a linear factor $(x - r)$ appears in the factorization, then the number *r* is an important *x*-value.)

Step 5: Make a sign chart. That is, put all the important *x*-values on a number line. In each region and at each important *x*-value, determine whether the function *f* is positive, negative, zero, or undefined.

Step 6: Locate vertical asymptotes, holes, and *x*-intercepts by examining the linear factors in the factorization. For each linear factor $(x - r)$ in the factorization, decide which of the following five cases applies.

case 1: The linear factor $(x - r)$ appears in the numerator but not in the denominator.

In this case, the graph will have an *x*-intercept at $x = r$.

case 2: The linear factor $(x - r)$ appears in the numerator and in the denominator but has a larger exponent in the numerator than it does in the denominator.

In this case, the graph will cross the *x*-axis at $x = r$, but there will be a hole at the crossing. (Technically, this is not called an "*x*-intercept", because $x = r$ is not in the domain.)

case 3: The linear factor $(x - r)$ appears in the numerator and in the denominator with equal exponents.

In this case, the graph will have a hole at $x = r$.

case 4: The linear factor $(x - r)$ appears in the numerator and in the denominator but has a smaller exponent in the numerator than it does in the denominator.

In this case, the graph will have a vertical asymptote at $x = r$.

case 5: The linear factor $(x - r)$ appears in the denominator only.

In this case, the graph will have a vertical asymptote at $x = r$.

Step 7: Based on the analysis in steps 1 through step 6, sketch the graph of f.