

Ten Step Method for Graphing Rational Functions with Calculus

Math 163A Calculus Section 01 (Barsamian)

Step 1: If $x = 0$ is in the domain, find $f(0)$.

Step 2: Check for symmetries.

Step 3: Determine the end behavior (horizontal asymptote? slant asymptote? power function?) by deciding which of the following three cases applies.

case 1: degree of numerator < degree of denominator

In this case, the end behavior will resemble $y = \frac{1}{x^m}$. So the line $y = 0$ will be a horizontal asymptote.

case 2: degree of numerator = degree of denominator

In this case, the end behavior will resemble $y = \frac{ax^m}{bx^m} = \frac{a}{b}x^0 = \frac{a}{b}$. So the line $y = \frac{a}{b}$ will be a horizontal asymptote.

case 3: degree of numerator > degree of denominator

case 3a: degree of numerator = 1 + degree of denominator

In this case, the end behavior will resemble $y = \frac{ax^{m+1}}{bx^m} = \frac{a}{b}x$. So, the line $y = \frac{a}{b}x$ will be a slant asymptote.

case 3b: degree of numerator ≥ 2 + degree of denominator

In this case, the end behavior will resemble $y = x^m$, for some integer $m \geq 2$.

Step 4: Make a sign chart for f . To do this, factor the function (both numerator and denominator). This will enable you to determine the domain of the function, and it will also tell you the important x -values for f . (If a linear factor $(x - r)$ appears in the factorization, then the number r is an important x -value.) Then, put all the important x -values on a number line. In each region and at each important x -value, determine whether f is positive, negative, zero, or undefined.

Step 5: Locate vertical asymptotes, holes, and x -intercepts by examining the linear factors in the factorization. For each linear factor $(x - r)$ in the factorization, decide which of the following five cases applies.

case 1: The linear factor $(x - r)$ appears in the numerator but not in the denominator.

In this case, the graph will have an x -intercept at $x = r$.

case 2: The linear factor $(x - r)$ appears in both the numerator and denominator but with a larger exponent in the numerator.

In this case, the graph will cross the x -axis at $x = r$, but there will be a hole at the crossing.

case 3: The linear factor $(x - r)$ appears in both the numerator and in the denominator and with equal exponents.

In this case, the graph will have a hole at $x = r$.

case 4: The linear factor $(x - r)$ appears in both the numerator and denominator but with a smaller exponent in the numerator.

In this case, the graph will have a vertical asymptote at $x = r$.

case 5: The linear factor $(x - r)$ appears in the denominator only.

In this case, the graph will have a vertical asymptote at $x = r$.

Step 6: Find the derivative f' and factor its numerator and denominator. This will tell you the important x -values for f' .

Step 7: Make a sign chart for f' to determine the x -values for which f' is positive, negative, zero, or undefined. This will tell you the x -values for which f is increasing, decreasing, or horizontal. The x -values for which $f'(x) = 0$ are called *critical numbers* of the function f . Plug the critical numbers into f to find the *critical values* and *critical points*.

Step 8: Find the second derivative f'' and factor its numerator and denominator. This will tell you the important x -values for f'' .

Step 9: Make a sign chart for f'' to determine the x -values for which f'' is positive, negative, zero, or undefined. This will tell you the x -values for which f is concave up, concave down, or not concave. The x -values at which the graph of f changes concavity (that is, where f'' changes sign) are called *points of inflection*. Plug these x -values into f to find the y -values of the points of inflection.

Step 10: Based on the analysis in steps 1 through step 9, sketch the graph of f .