Worksheet for the "D-test"

Goal: find relative extrema & saddle points in a function $f(x,y)$.

1. Find $f_x(x,y)$.
2. Find $f_y(x,y)$.
3. Find $f_{xx}(x,y)$.
4. Find $f_{yy}(x,y)$.
5. Find $f_{xy}(x,y)$.
6. Build $D(x,y)$. Remember, $D(x,y) = f_{xx}(x,y) \cdot f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$.
7. Find the critical points. (These would better be called "critical pairs"). They are the pairs (a,b) such that $f_x(a,b) = 0$ and $f_y(a,b) = 0$. To find them, solve this pair of equations:
$$\begin{cases} Equation\ A:\ f_x(x,y) = 0 \\ Equation\ B:\ f_y(x,y) = 0 \end{cases}$$
8. For each critical pair (a,b), compute $D(a,b)$ and note whether it is positive, negative, or zero. If a critical pair (a,b) causes $D(a,b)$ to be negative, then you can immediately conclude that an input of (a,b) will cause a saddle point in the graph of f. If a critical pair (a,b) causes $D(a,b)$ to be zero, then you can immediately conclude that you are out of luck: the "D-test" does not tell you what the input (a,b) will cause in the graph of f. If a critical pair (a,b) causes $D(a,b)$ to be positive, then an input of (a,b) will cause either a relative max or a relative min in the graph of f. The next step will tell you which.
9. For each critical pair (a,b) that causes $D(a,b)$ to be positive, compute $f_{xx}(a,b)$. If $f_{xx}(a,b)$ is positive, then an input of (a,b) will cause a relative min in the graph of f. If $f_{xx}(a,b)$ is negative, then an input of (a,b) will cause a relative max in the graph of f.
10. Write your conclusions about the critical points clearly.