

Worksheet for the method of Lagrange Multipliers

Goal: find a max or a minimum in some equation $f(x, y)$ subject to some constraint of the form $g(x, y) = 0$.

1. Identify the function to be maximized, $f(x, y)$.

Identify the constraint function, $g(x, y)$.

2. Find $f_x(x, y)$.

3. Find $f_y(x, y)$.

4. Find $g_x(x, y)$.

5. Find $g_y(x, y)$.

6. Build equation A: $f_x(x, y) = \lambda g_x(x, y)$.

7. Build equation B: $f_y(x, y) = \lambda g_y(x, y)$.

8. Build equation C: $g(x, y) = 0$.

9. Solve the system of three equations, A, B, C. (Try to start by solving equation C for one variable in terms of the other.) The result will be one or more distinguished pairs of the form (a, b) .

10. If the function $f(x, y)$ (with the given constraint) has any relative max's or min's, they will occur when one of the distinguished pairs is used as inputs. For each distinguished pair (a, b) , decide whether the pair is a max or a min in the following way. Suppose that you were able to solve equation C for y in terms of x . You should see what happens to the output when you use x values equal to a and also slightly lower and slightly higher than a . Because of the constraint equation, you will have to get the corresponding values of y from equation C. Plug each (x, y) pair into f to get a value for the resulting output.

value to use for x	value to use for y	value of $f(x, y)$
number slightly lower than a	number obtained from equation C	?
a	b	?
number slightly higher than a	number obtained from equation C	?

Alternately, if you had to solve equation C for x in terms of y , then you should see what happens to the output when you use y values equal to b and also slightly higher and slightly lower than b . Because of the constraint equation, you will have to get the corresponding values of x from equation C.

value to use for x	value to use for y	value of $f(x, y)$
number obtained from equation C	number slightly lower than b	?
a	b	?
number obtained from equation C	number slightly higher than b	?

Now compare the resulting outputs. If the output in the middle row of the table is the largest of the three outputs, then it is a safe guess that the pair (a, b) corresponds to a maximum. If the output in the middle row of the table is the smallest of the three outputs, then it is a safe guess that the pair (a, b) corresponds to a minimum. For each pair (a, b) , make a table, then use the table to decide whether (a, b) is a max or a min.