Handout 9: Worksheet for the "D-test"

Fall 2006 Math 163A Calculus Section 02 (Barsamian)

Goal: find relative extrema & saddle points in a function $f(x,y)$.

1. Find $f_x(x,y)$.

2. Find $f_y(x,y)$.

3. Find the "mixed" second order partial derivatives.
   a. Find $f_{xy}(x,y)$.
   b. Find $f_{yx}(x,y)$.
   c. Compare your answers to 3a and 3b. If they are not the same, then check your work in steps 1, 2, 3a, and 3b for errors. If your answers to 3a and 3b are the same, go on to step 4.

4. Find $f_{xx}(x,y)$.

5. Find $f_{yy}(x,y)$.

6. Build $D(x,y)$. Remember, $D(x,y) = f_{xx}(x,y) \cdot f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$.

7. Find the critical points. (These would better be called "critical pairs"). They are the pairs (a,b) such that $f_x(a,b) = 0$ and $f_y(a,b) = 0$. To find them, solve this pair of equations:
$$\begin{cases} Equation\ A: \ f_x(x,y) = 0 \\ Equation\ B: \ f_y(x,y) = 0 \end{cases}$$

8. For each critical pair (a,b), compute $D(a,b)$ and note whether it is positive, negative, or zero. If a critical pair (a,b) causes $D(a,b)$ to be negative, then you can immediately conclude that an input of (a,b) will cause a saddle point in the graph of f. If a critical pair (a,b) causes $D(a,b)$ to be zero, then you can immediately conclude that you are out of luck: the "D-test" does not tell you what the input (a,b) will cause in the graph of f. If a critical pair (a,b) causes $D(a,b)$ to be positive, then an input of (a,b) will cause either a relative max or a relative min in the graph of f. The next step will tell you which.

9. For each critical pair (a,b) that causes $D(a,b)$ to be positive, compute $f_{xx}(a,b)$. If $f_{xx}(a,b)$ is positive, then an input of (a,b) will cause a relative min in the graph of f. If $f_{xx}(a,b)$ is negative, then an input of (a,b) will cause a relative max in the graph of f.

10. Write your conclusions about the critical points clearly.