

## Solutions to Math 266A Group Work 4: Graphing a Cubic Polynomial

Thursday, March 02, 2006

Find the important points on the graph of the function  $f(x) = x^3 - 3x^2 - 9x + 27$ , and then graph it.

Hint: Think of the following list of questions, which starts easy and then gets harder.

Difficulty	Question	How do you answer it?
Easy	What is the y-intercept?	Call 1-800-ynt-rcpt and ask for Vinny.
Pretty easy.	What is the end behavior?	Consider degree & leading coefficient.
Not so easy.	When is $f$ positive, negative, or zero?	Factor $f$ and make a sign chart for it.
Getting hard.	When is $f$ increasing, decreasing, or horizontal?	Factor $f'$ and make a sign chart for it.
Crazy hard.	When is $f$ concave up, down, or inflecting?	Factor $f''$ and make a sign chart for it.

Hint: the function can be factored  $f(x) = x^3 - 3x^2 - 9x + 27 = (x+3)\left[(x-3)^2\right]$ .

### Step 1: Find the y-intercept

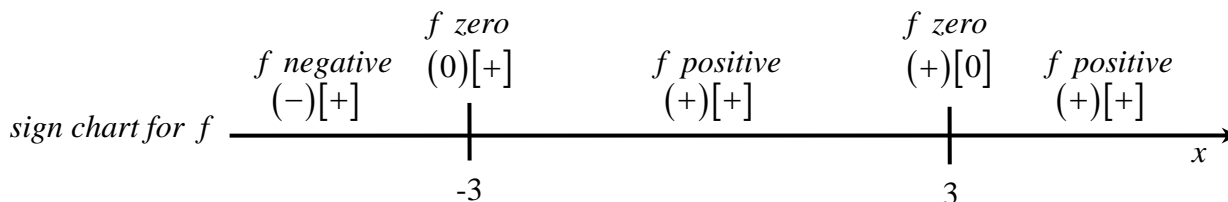
$f(0) = (0)^3 - 3(0)^2 - 9(0) + 27 = 27$ , so the point  $(0, 27)$  is the y-intercept.

### Step 2: Determine the end behavior

The function  $f$  is a cubic polynomial, so one side of its graph will go up and the other will go down. Because the leading coefficient is positive, we know that the graph will go up on the right and down on the left.

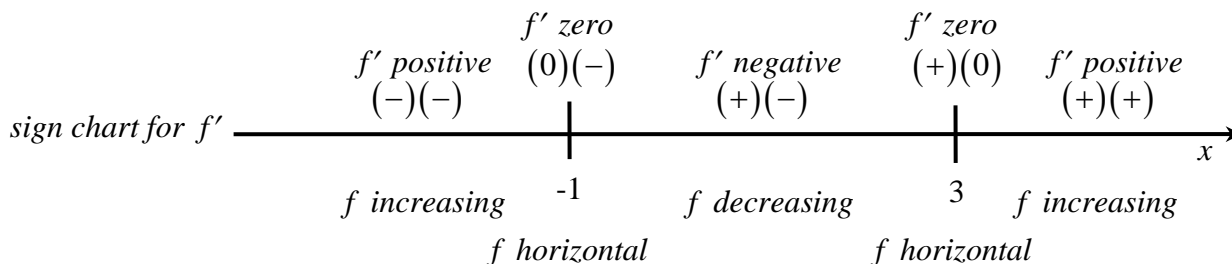
### Step 3: Determine when $f$ is positive, negative, and zero

We know that the function  $f$  factors as  $f(x) = x^3 - 3x^2 - 9x + 27 = (x+3)\left[(x-3)^2\right]$ . To determine when  $f$  is positive, negative, and zero, we make a sign chart for  $f$ . On the sign chart, we have to remember that the factor that is inside the square brackets will never be negative. It will only be zero or positive. The sign chart for  $f$  is:



### Step 4: Determine when $f$ is increasing, decreasing and horizontal

To determine when  $f$  is positive, negative, and zero, we have to find out when  $f'$  is positive, negative, and zero. To do that, we make a sign chart for  $f'$ . We first need to find  $f'$ . Computing the derivative and factoring, we find  $f'(x) = 3x^2 - 6x - 9 = 3(x+1)(x-3)$ . The sign chart for  $f'$  is:

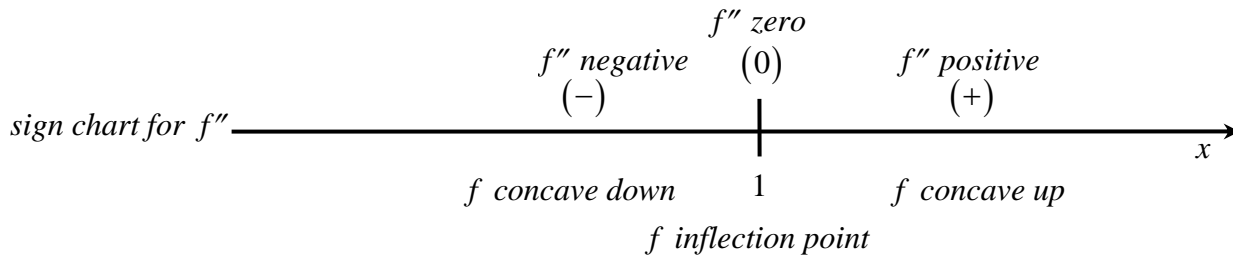


We see that  $f$  changes from increasing to decreasing at  $x = -1$ . Therefore,  $x = -1$  is the location of a relative maximum. We should get the corresponding  $y$ -value so that we can make a really great graph. The value is  $f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 27 = 32$ . So the point  $(-1, 32)$  is a relative max.

Also note that  $f$  changes from decreasing to increasing at  $x = 3$ . Therefore,  $x = 3$  is the location of a relative minimum. We already know from step 3 that at  $x = 3$ , the corresponding  $y$ -value is zero. So the point  $(3, 0)$  is a relative min.

**Step 5: Determine when  $f$  is concave up, concave down, and inflecting**

To determine when  $f$  is concave up, concave down, and inflecting, we have to find out when  $f''$  is positive, negative, and zero. Do do that, we make a sign chart for  $f''$ . We first need to find  $f''$ . Computing the derivative and factoring, we find  $f''(x) = 6x - 6 = 6(x - 1)$ . The sign chart for  $f''$  is:



We see that  $f$  has an inflection point at  $x = 1$ . The  $y$ -value is  $f(1) = (1)^3 - 3(1)^2 - 9(1) + 27 = 16$ . So the point  $(1, 16)$  is an inflection point.

Here is the graph:

