MATLAB 4: Graph for Exercise 4.3#71

In exercise 4.3#71 from Homework 4, you are asked to find the equation for the line tangent to the graph of $f(x)=\frac{x^2+3}{x^3+5}$ at the point $x=-2$. You will use MATLAB to check your work and to make a graph of the function and the tangent line. You will learn some new basic MATLAB skills, including:

- getting MATLAB to display results as fractions instead of as decimals
- getting MATLAB to simplify results
- using MATLAB to differentiate a function
- dealing with unpredictable MATLAB graph behavior
- plotting and highlighting single points
- plotting horizontal and vertical lines

Start by doing this preliminary work:

1) Solve exercise 4.3#71 from Homework 4. Show all your steps. Do not use decimals: work with fractions. Make sure that your write-up includes the following things:
   a) the value of $f(-2)$, written as a fraction
   b) the coordinates of the known point $(-2, f(-2))$, written as fractions
   c) the value of $f'(x)$, simplified as best you can
   d) the value of $f'(-2)$
   e) the point-slope form of the equation of the tangent line, using fractions
   f) the equation of the tangent line, rearranged to slope-intercept form, using fractions
2) Compare your answer to the answer in the back of the textbook. The book's answer might be wrong.

Now begin the MATLAB work:

3) Start the MATLAB program. You should see the *command window*, with the *command prompt* >>.
4) Type `>>clear` Enter.
5) Type `>>syms x` Enter
6) Type `>>f=(x^2+3)/(x^3+5)` Enter

You now will use MATLAB to check the value that you got for $f(-2)$ in your solution to 4.3#71.

Method 1:

7) Type `>>subs(f,-2)` Enter

Method 2:

8) Type `>>subs(f,'-2')` Enter
9) Type `>>simplify(ans)` Enter
10) Question: How do the results of Method 1 and Method 2 differ?
11) Question: Is the value that you got for $f(-2)$ in your solution to 4.3#71 correct?

You will now use MATLAB to differentiate f. The derivative will be denoted `fprime`.

12) Type `>>fprime=diff(f)` Enter
13) Type `>>simplify(fprime)` Enter
14) Question: Is the value that you got for $f'(x)$ in your solution to 4.3#71 correct?
15) Type `>>subs(fprime,-2)` Enter
16) Question: Is the value that you got for $f'(-2)$ in your solution to 4.3#71 correct?

You will now try two different ways of graphing the function f.
Method 1:
17) Type `>>X=-5:.1:5` Enter
18) Type `>>F=subs(f,X)` Enter (Mind the upper and lower case letters.)
19) Type `>>plot(X,F)` Enter
The graph is supposed to have a vertical asymptote, but MATLAB is messing up the graph by trying to connect the left and right pieces of the graph.
20) Click on the "x" at the upper right corner of the plot window to make this graph go away.
Method 2:
21) Type `>>ezplot(f,-5,5)` Enter (Mind the lower case letter.)
This graph will come out better, but I don't know why. Sometimes the "plot" command works better than the "ezplot" command; sometimes "ezplot" works better. The MATLAB documentation does not explain the quirks of these commands. The moral is that when you use MATLAB on your own, you will often have to try plotting functions in more than one way, because of unexplained wierd MATLAB behavior.

Now you will graph the tangent line.
22) Type `>>hold on` Enter
23) Type `>>g=-8*x-55/3` Enter This is the equation for the line tangent to the graph of f at $x=-2$.
24) Question: Is the equation for the tangent line in your solution to 4.3#71 correct?
25) Type `>>ezplot(g,-5,5)` Enter
26) Change the scale of the axes to show $-5 \le x \le 5$ and $-5 \le y \le 5$.

You will now plot a single point at the intersection of the graphs of f and g.
27) Type `>>plot(-2,-7/3)` Enter You will not see the resulting point, because it is being covered up by the thick graphs of f and g.
28) Turn the graphs of f and g yellow. This should allow you to see the single point that you plotted.
29) Using the arrow tool, click on the single point.
30) Change the single point so that it has a round marker of size 15.0, colored black.
31) Change the graph of f so that it is a black line of thickness 2.0.
32) Change the graph of g so that it is a red line of thickness 2.0.

You will now add horizontal and vertical asymptotes to your picture.
The horizontal asymptote is easy, because it is described by the constant function $y=0$.
33) Type `>>ezplot(0,-5,5)` Enter The horizontal asymptote should appear.
The vertical asymptote is harder, because vertical lines are not described by functions. Our vertical asymptote is described by the equation $x=-\sqrt[3]{5}$. The command that we use to draw this line is rather strange.
34) Type `>>plot(-5^(1/3)*[1 1],[-5 5])` Enter Note that there is a space in the `[1 1]` and another space in the `[-5 5]`. The vertical asymptote should appear.
35) Change the asymptotes so that they are black dotted lines of thickness 2.0.

Your picture now includes the graph of f, the tangent line, a dot at their intersection point, and two asymptotes.
36) Using MATLAB, label all of these items, and add the usual title (your name, the exercise number, etc).
37) Print your graph.
38) You should have your solution to 4.3#71 and answers to MATLAB questions 10), 11), 14), 16), and 24) written down on a piece of paper. Attach that piece of paper and your graph to your Homework 4 solutions.