Math 330A Homework Set #3, Due Friday, October 10, 2008

[1] The goal is to solve problem 1.3#6abcd from the book. I will give you some additional hints.
(a) For part (a) of problem #6 in the book, find an equation that gives *x* in terms of *y* and *z*. (Hint: Through the point that is the vertex of angle *y*, draw a line that is parallel to line *a*. Look at the way this line breaks up *y*.)
(b) For part (b) of problem #6 in the book, find an equation that gives α_4 in terms of α_3, α_2, and α_1. (Hint: Through the point that is the vertex of angle α_3, draw a line that is parallel to line *a*. Look at the way this line breaks up angle α_3. Use the result that you found in part (a) to save steps.)
(c) For part (ci) of problem #6 in the book, draw a new picture that has five angles. Find an equation that gives α_5 in terms of α_4, α_3, α_2, and α_1. (Hint: Through the point that is the vertex of angle α_4, draw a line that is parallel to line *a*. Notice how this line breaks up angle α_4. Use the result that you found in (b) to save steps.)
(d) For part (d) of the problem in the book, notice the pattern from parts (a), (b), and (c).

[2] This is a variation of problem 1.3#9 in the book.
(a) Find a formula that expresses the measure of angle *D* in terms of the measure of Angle *A*.
(b) Create a Geogebra drawing that confirms that your formula is correct. In your drawing, $\triangle ABC$ should be free and there should be measurements to confirm that your formula is correct. Somewhere in your drawing, put a text box with the text "H3.2.Lastname". Save the file with the filename "H3.2.Lastname".

[3] Problem 1.3#11 in the book has diagrams suggesting a strategy for proving Theorem 1.13. Write a proof of the theorem. (Hint: I did this proof in class last week! I called it the BS ➔ BA theorem.)

[4] Here is a statement of the Midsegment Theorem for triangles, and a proof. Some of the steps in the proof have not been justified. Justify them by citing axioms or prior theorems. Make drawings where indicated.
Theorem 1.31(Triangle Midsegment Theorem) In Euclidean Geometry, in any triangle, the segment connecting the midpoints of two sides of the triangle is parallel to the third side and is half as long as the third side.
Proof

1. In Euclidean Geometry, suppose that $\triangle ABC$ is a triangle, point *M* is the midpoint of side $\overline{AB}$, and point *N* is the midpoint of side $\overline{AC}$. (Make a drawing showing all information given in this statement.)

Part 1: Find two triangles that are congruent.

2. There exists a point P on line $\overleftrightarrow{MN}$ such that *M-N-P* and such that *NM* = *NP*. (justify)
3. $\angle ANM \cong \angle CNP$. (Justify)
4. $\triangle ANM \cong \triangle CNP$. (Justify) (Make a new drawing showing all information known at this point.)

Part 2: Identify a parallelogram

5. $\angle AMN \cong \angle CPN$. (Justify)
6. Segment $\overline{BM}$ is parallel to segment $\overline{CP}$. (Justify)
7. Segment $\overline{BM} \cong \overline{CP}$. (Justify)
8. Quadrilateral *BCPM* is a parallelogram. (Justify)

Part 3: Prove stuff about segment $\overline{MN}$.

9. Segment $\overline{MN}$ is parallel to segment $\overline{BC}$. (Justify)
10. *MN* = (1/2)*BC*. (Justify)

End of Proof.

[5] (a) Prove this Theorem: In Euclidean geometry, for any quadrilateral *ABCD*, with points *M*, *N*, *P*, *Q* the midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, the resulting quadrilateral *MNPQ* is a parallelogram.
Hint: Draw diagonal $\overline{AC}$ to create two triangles. Apply the Midsegment Theorem to both triangles.
(b) Create a Geogebra drawing showing free quadrilateral ABCD and dependent quadrilateral MNPQ. Add measurements to your drawing that confirm that quadrilateral MNPQ is a parallelogram. Somewhere in your drawing, put a text box with the text "H3.5.Lastname". Save the file with the filename "H3.5.Lastname."

[6] Do problem 1.3#19.

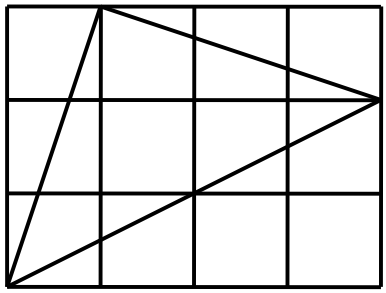
[7] (a) Do problem 1.3#28. Hint: Draw a segment from point *M* to the origin (where the right angle is located.) (b) Make a Geogebra drawing that illustrates your answer. In your drawing, point *B* should be free to move on the horizontal axis and the segment $\overline{AB}$ should have fixed length 2 units. Point *A* should be dependent, but it should be constrained somehow so that it is allowed to move only up and down, along the vertical axis. Add measurements to your drawing that confirm your answer. Somewhere in your drawing, put a text box with the text "H3.7.Lastname". Save the file with the filename "H3.7.Lastname."

[8] Do problem 1.3#31.

Hint: First, explain why $z = 45^\circ$.

Then, use the picture at right to show that $x + y = 45^\circ$. You will need to

- show that the triangle is isosceles
- show that the triangle is a right triangle
- show that the quantity x + y is the measure of one of the angles.
- show that the measure of that angle is 45.



[9] The goal of this problem is to solve the Treasure Island Problem using the outline suggested in the "Now Solve This 1.13" on page 55-56. I will fill in some of the details and outline the proof steps more explicitly.

- Let point *G* be the gallows (The book uses a Greek letter; but *G* is simpler), let points T_1 and T_2 be the two trees, let points S_1 and S_2 be the two spikes, and let point *M* be the treasure, all as described in the Treasure Island Problem and as shown in figure 1.61 on page 56.
- Let *A, B, C, N* be the points at the feet of the perpendiculars drawn from points S_1, S_2, *G*, *M* to line $\overleftrightarrow{T_1T_2}$.

Part 1 Identify two congruent triangles and some consequences

- Prove that $\triangle GT_1C \cong \triangle T_1S_1A$.
- Prove that $GC = T_1A$. Call this length *h*.
- Prove that $T_1C = S_1A$. Call this length *a*.

Part 2 Identify two more congruent triangles and some consequences

- Prove that $\triangle GT_2C \cong \triangle T_2S_2A$.
- Prove that $GC = T_2B$. This is length *h* again.
- Prove that $T_2C = S_2B$. Call this length *b*.

Part 3 Use the Trapezoid Midsegment Theorem

- Prove that $NA = NB$.
- Prove that $MN = (1/2)(a + b)$ (Hint: Segment $\overline{MN}$ is the midsegment of a trapezoid. Identify the trapezoid and then use the Trapezoid Midsegment Theorem.)

Part 4: Wrap-up

- Prove that $MN = (1/2)T_1T_2$.
- Prove that $NT_1 = NT_2$.

[10] You created Geogebra drawings for problems [2], [5], and [7] above. Put the three drawings into a folder called "H3.Lastname". Create a zipped version of this folder (either using the 7-zip command or some other program). E-mail this zipped folder.

- recipients:
  - me: Mark.Barsamian.1@ohio.edu
  - you: use your OU e-mail address
- Subject line: "Math 330A H3.Lastname"
- Attachment: the zipped folder called "H3.Lastname"