Math 330A/539 Homework Set #4, Due Thursday, October 23, 2008

Preliminary work: Create a folder called "H4.Lastname" on your computer desktop.

[1] Here is a theorem that combines a simplified version of Proposition 2.3 and its converse.

Theorem: In a circle with chord AB and diameter CD, the following statements are equivalent:

(1) CD is perpendicular to AB.

(2) CD bisects AB.

Prove the Theorem without using any of Propositions, Theorems, or other results of Chapter 2.
Hint 1: Remember proof structure: Your job is to prove that (1) ➔ (2) and that (2) ➔ (1)
Hint 2: Let O be the center of the circle. Draw segments OA and OB. Consider the resulting triangles.

[2] Theorem 2.3 on page 82 of the textbook is so badly worded and contains so many editing mistakes that it is almost impossible to understand what the Theorem is trying to say. But underneath the mess is a very cool theorem.

Remember this Background Information: It is a fact of Euclidean Geometry that given any three non-collinear points A, B, C, there exists exactly one circle that contains all three points.

Here is a reworded version of the theorem:

Reworded Theorem 2.3 Given three non-collinear points A, B, C and a point P on the same side of line BC as point A, the following statements are equivalent:

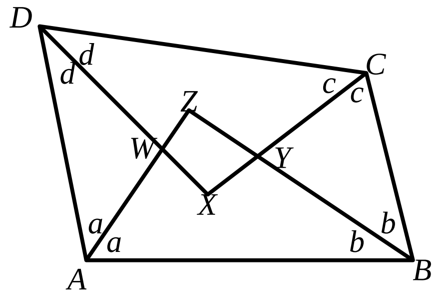
(1) angle BPC is congruent to angle BAC

(2) Point P is on arc BAC of the circle containing points A, B, C.

(a) Prove Theorem 2.3
Big Hint: First prove that (2) ➔ (1). This should be easy. Next, prove that ~(2) ➔ ~(1). To do this, you start by suppose that ~(2) is true. That is, you suppose that point P is not on arc ABC. But you still know the given information that point P is on the same side of line BC as point A. There are two possible cases to consider: Point P could be inside the circle or outside the circle. In each case, the line BP intersects the circle at point B and at another point that can be called Q. Consider the relative sizes of angle BAC, angle BQC, and angle BPC. It will be very useful to apply the Exterior Angle Theorem to triangle CPQ, angle BQC, and angle BPC.

(b) Create a Geogebra drawing to illustrate the theorem. Your drawing should have the following free things: Points A, B, C, and P. The drawing should have the following dependent things: Line BC; the circle containing points A, B, and C; and Rays AB, AC, PA, and PC. It should also show the measure of angles BAC and BPC. Save your drawing with the filename "H4.2.Lastname" in the folder called "H4.Lastname" on your computer desktop.

[3] This problem is based on the "Now Solve This 2.5.2" on pages 86-87 of the textbook.
(a) Prove the following Theorem:



If ABCD and WXYZ are quadrilaterals such that

- *A-W-Z and line AZ bisects angle A,*
- *B-Y-Z and line BZ bisects angle B,*
- *C-Y-X and line CX bisects angle C, and*
- *D-W-X and line DX bisects angle A,*

then quadrilateral WXYZ is cyclic.

Remark: Don't ask me what cyclic means. Read the book.

Hint: Let the symbol little *a* denote the measure of the each of the two small angles created by the bisector of angle A. Similarly for the symbols little *b*, *c*, and *d*. Get an equation involving the four symbols a,b,c,d. Get another equation involving the symbols a, b, and Z. Get another equation involving the symbols c, d, and X. Using these three equations, you should be able to get a value for X + Z.

(b) Create a Geogebra drawing that illustrates the theorem. In your drawing, points A, B, C, and D should be free, and everything else should be dependent. Your drawing should include the circle determined by the three points W,X,Y. Save your drawing with the filename "H4.3.Lastname" in the folder called "H4.Lastname" on your computer desktop.

[4] This exercise is based on the "Now Solve This 2.1" on page 78, and on exercise 2.1#3.

First, some remarks on notation.

- The symbol "Circle(A,AP)" will stand for the circle centered at A with radial segment AP.
- The book defines a "tangent to a circle" to be a line that is perpendicular to a radial segment of the circle at the point of the radial segment that lies on the circle.
- Most books define a "tangent to a circle" to be a line that intersects the circle exactly once.
- The moral is, you have to be careful to check the definitions of terms in whatever book you are reading.
- Because of the potential confusion involving the phrase "tangent to a circle", I will not use the phrase in this course. If you use it in class or in your writing, you should always be sure to remind the listener or the reader what you mean by it.

This problem is continued on the next page.

Problem [4] continued.

The following theorem is famous. Justify the steps in the proof. Make drawings where indicated.

Theorem: In Neutral Geometry, the following are equivalent:

(1) Line *L* is perpendicular to segment AP at point *P*.

(2) Line *L* intersects Circle(A,AP) exactly once, at point *P*.

Proof part 1: Show that (1) ➔ (2)

1. Suppose that Line L is perpendicular to segment AP at point P.
2. Line *L* intersects Circle(A,AP) at point *P*. (by step 1 and definition of perpendicular)
3. Assume that line *L* intersects Circle(A,AP) at another point *Q*. (assumption) (make a drawing)
4. Segment AP is congruent to segment AQ (justify)
5. Angle AQP is congruent to angle APQ (justify)
6. Lines AQ and AP are parallel (justify)
7. Step 6 contradicts the fact that lines AQ and AP intersect at point *A*. So our assumption in step (3) must be wrong. Conclude that Line L intersects Circle(A,AP) exactly once, at point P.

End of proof part 1

Proof part 2: Show that (2) ➔ (1)

1. Suppose that Line L intersects Circle(A,AP) exactly once, at point P.
2. Assume that line *L* is not perpendicular to segment AP at point *P*. (assumption)
3. There exists a line *M* that passes through point *A* that is perpendicular to line *L*. *(justify)* The point of intersection of lines *M* and *L* cannot be point *P* (by step 2), so we can give the intersection the label *Q*. So line *M* is line AQ. (make a drawing)
4. Observe that Angle AQP measures 90 degrees. (by steps 3 and 4 and definition of perpendicular)
5. There exists a point *R* such that *P-Q-R* and such that QP = QR (justify)
6. Angle AQR measures 90 degrees (justify)
7. Segment AQ is congruent to itself. (justify)
8. Triangle AQP is congruent to triangle AQR (justify)
9. Segement AP is congruent to segment AR (justify)
10. Point *R* is on Circle(A,AP) (justify)
11. Line *L* intersects Circle(A,AP) at point *R*. (by step 11)
12. Step 12 contradicts the fact that line *L* only intersects the circle at *P*. Therefore, our assumption in step 2 was wrong. It cannot be that line *L* is not perpendicular to segment AP at point *P*. Conclude line *L* is perpendicular to segment AP at point *P*.

End of proof

[5] Do problem 2.1#7. Note that the answers to this problem are in the back of the book. Your job is to justify those answers.

[6] Do problem 2.1#9a in the book.

[7] (a) Do problem 2.1#15 in the book.
(b) Make a Geogebra drawing that illustrates the problem. Here is a suggestion for how to make the drawing.

- Create points A and B. These should both be free.
- Create point O as the midpoint between A and B.
- Create Circle(O,OB).
- Create Line AB
- Create a point C that lies on the circle. (Point C should be free to move around the circle.)
- Create segment OC.
- Create a line L through C perpendicular to OC.
- Create a point S at the intersection of line L and Line AB
- Create the bisector of angle ASC. Call the bisector line M.
- Create the point T where line M intersects segment OC.
- Measure Angle STC.

Save your drawing in a file called “H4.7.Lastname” in the folder called “H4.Lastname” on your computer desktop

[8] You created Geogebra drawings for problems [2], [3], and [7] and saved them in the folder called “H4.Lastname”. Create a zipped version of this folder (either using the 7-zip command or some other program). E-mail this zipped folder.

- recipients:
    - me: Mark.Barsamian.1@ohio.edu
    - you: use your OU e-mail address
- Subject line: “Math 330A H4.Lastname”
- Attachment: the zipped folder called “H4.Lastname”