Math 330A (Barsamian) Computer Project 3: The Alternate Interior Angle Theorem

Concept Review: Conditional Statements and their Converses

Remember from Math 306 the definition of the converse of a conditional statement:

	in words	*in symbols*
Statement S:	*If Pthen Q*	$P \rightarrow Q$
Converse of Statement S:	*If Qthen P*	$Q \rightarrow P$

It is very important to remember Statement *S* and the Converse of Statement *S* do not mean the same thing. They are *not* logically equivalent. The fact that some conditional statement *S* is true or false has no bearing at all on the issue of whether the *converse of S* is true or false.

Today, we will be making drawings to illustrate the following conditional statement and its converse.

Statement S:	*If a pair of alternate interior angles is congruent, then lines L and M are parallel*
Converse of S:	*If lines L and M are parallel, then a pair of alternate interior angles is congruent.*

Concept Review: The convention for names of theorems in math

As I mentioned in class on Thursday, May 8, when a theorem is given a name, the name usually refers to the given information and the hypotheses. For example consider the following theorem:

Theorem: In Neutral Geometry, If a hippo is green then the hippo has three legs.

This theorem is telling us something about green hippos in Neutral geometry. (It is telling us that they all have three legs.) According to the naming convention, this would be called the *"Green Hippo Theorem of Neutral Geometry"*. It would *not* be called the *"Three-legged Hippo Theorem of Neutral Geometry"*. (This sort of makes sense. The theorem does not tell us anything about three-legged hippos.)

Theorem Review: The Alternate Interior Angle Theorem

Recall the statement of the Alternate Interior Angle Theorem:

Theorem 38: The Alternate Interior Angle Theorem
Given: Neutral Geometry, lines L and M and a transversal T
Claim: If a pair of alternate interior angles is congruent, then lines L and M are parallel.

Notice that this theorem tells us something about any situation where a pair of alternate interior angles are congruent. (It tells us that the lines are parallel.) So it makes sense that the theorem should be named the Alternate Interior Angle Theorem. Remember that the statement of the Alternate Interior Angle Theorem is *true*, because we have proved that it is true. That's why it's a theorem. The first part of today's lab will be to make drawings to illustrate the truth of the statement of the Alternate Interior Angle Theorem.

What about the converse statement?

Consider the following statement:

The Converse of The Alternate Interior Angle Theorem
Given: Neutral Geometry, lines L and M and a transversal T
Claim: If lines L and M are parallel, then a pair of alternate interior angles is congruent.

Is the converse statement true? We don't know yet. The second part of today's lab will be to make drawings to explore the truth or falseness of the Converse of the Alternate Interior Angle Theorem

Computer login & filing tasks

1. Login to the computer
   - id: your Oak ID
   - password: last 6 digits of your student ID number
2. On your computer desktop, create a folder called CP3.

Part I: Drawings to illustrate the Alternate Interior Angle Theorem

Geometer's Sketchpad Tasks (Drawings of Euclidean Geometry)

3. Open a new drawing in Geometer's SketchPad (GSP)
4. Create a line $\overleftrightarrow{AB}$.
5. Give line $\overleftrightarrow{AB}$ the label *L*.
6. Create another line $\overleftrightarrow{BC}$ that intersects line *L* at point *B*.
7. Give line $\overleftrightarrow{BC}$ the label *T*.

The goal is to create a line $\overleftrightarrow{CD}$ such that

- points *D* and *A* are on opposite sides of line line $\overleftrightarrow{BC}$.
- $\measuredangle DCB \cong \measuredangle ABC$

But we would like to do this in a way that when point *C* is moved around, everything gets automatically adjusted in a way that $\measuredangle DCB$ remains congruent to $\measuredangle ABC$. In *GSP*, the way to do this is to create point *D* by rotating point *B* around point *C* through an angle determined by the size of $\measuredangle ABC$.

We start by giving *GSP* the center of rotation.

8. With the selection tool (the arrow), unselect everything, then select point *C*.
9. On the [Transform] menu, click [mark center]

Next, we give GSP the angle of rotation

10. With the selection tool, unselect everything, then select the set of three points *C*,*B*,*A* in that order. (Leave *C* selected while selecting *B*, and leave *C* and *B* selected while selecting *A*.)
11. On the [Transform] menu, click [mark angle]

Next, we tell *GSP* what objects we want to be rotated.

12. With the selection tool, unselect everything, then select point *B*.
13. On the [Transform] menu, click [rotate]. A little pop-up window will appear, telling you that point *B* is going to be rotated about center point *C*, and giving you the option of rotating by a fixed angle or by a marked angle. The marked angle is angle $\measuredangle CBA$.
14. Be sure that the marked angle option is selected.
15. Click [rotate] to tell *GSP* to go ahead and rotate point *B*. It will create a new point called *B'*.
16. Relabel point *B'* as point *D*.
17. Create line $\overleftrightarrow{CD}$.
18. Give line $\overleftrightarrow{CD}$ the label M.

Measure stuff

19. Measure $\measuredangle ABC$ and $\measuredangle DCB$.

Wrap up.

Your drawing should now show the lines *L* and *M* and transversal *T*. The two angles $\measuredangle DCB$ and $\measuredangle ABC$ are congruent alternate interior angles. Their measurements should be displayed. Lines *L* and *M* should look parallel.

20. Save a picture of your drawing in its current state.
    - On the GSP [File] menu, click [Save As...]
    - Save in: CP3 folder that you created on the desktop
    - File name: drawing 1
    - Save as type: Windows Metafile (*.wmf)
21. Back in your drawing in GSP, move point C (drastically).
22. Yell "Eureka!" (Your drawings illustrate the Alternate Interior Angle Theorem: You created Alternate Interior Angles that are congruent, and as a result, lines *L* and *M* are parallel. As you move point *C* around, the alternate interior angles remain congruent (because of the way you constructed your drawing) and as a result, the lines remain parallel.)
23. Save another image of your drawing in its current state.
    - File name: drawing 2

Microsoft Word Tasks

24. Open a new document in Microsoft Word (*MSWord*)
25. Create a title in the document
    - Your Name
    - Spring 2008 Math 330A Computer Project 3
    - Exploring the Alternate Interior Angle Theorem and its Converse
26. Save the document
    - Save in : CP3 folder that you created on the desktop
    - Filename: 330A_CP3_YourLastName
    - File type: .doc
27. Insert into your document the four drawings that you saved from *GSP*.
28. Crop and re-size the drawings so that they look good. Try to get two drawings on one page, but in a way that the labels are still legible.
29. Type captions and brief explanations for each drawing.

NonEuclid Tasks (Drawings of the Poincare Disk)

30. Open a new drawing in *NonEuclid.*
31. Create a line $\overleftrightarrow{AB}$.
32. Create another line $\overleftrightarrow{BC}$ that intersects line $\overleftrightarrow{AB}$ at point *B*.

The goal is to create a line $\overleftrightarrow{CF}$ such that

- points *F* and *A* are on opposite sides of line $\overleftrightarrow{BC}$.
- $\measuredangle FCB \cong \measuredangle ABC$

But we would like to do this in a way that when point *C* is moved around, everything gets automatically adjusted in a way that $\measuredangle FCB$ remains congruent to $\measuredangle ABC$. In Non-Euclid, the way to do this is to construct point *F* by reflecting point *A* across some special lines that we will create.

33. Construct the midpoint of segment $\overline{BC}$. Non-Euclid will label the midpoint as *D*.
34. Create line $\overleftrightarrow{AD}$.
35. Construct a line that passes through *D* and is perpendicular to $\overleftrightarrow{AD}$. Non-Euclid will create a line $\overleftrightarrow{DE}$.
36. Reflect point *A* across line $\overleftrightarrow{DE}$. Non-Euclid will create a point *F* on line $\overleftrightarrow{AD}$.
37. Create line $\overleftrightarrow{CF}$.

Observe some interesting stuff:

- $\overline{CD} \cong \overline{BD}$ (because *D* is the midpoint of $\overline{BC}$)
- $\measuredangle FDC \cong \measuredangle ADB$ (because they are vertical angles)
- $\overline{FD} \cong \overline{AD}$ (because of the way that point *F* was created by reflecing point *A*.)
- Therefore, $\triangle CDF \cong \triangle BDA$ (by CA6)
- Therefore, $\measuredangle FCD \cong \measuredangle ABD$ (by definition of triangle congruence, that is, by *CPCTC*)

So we have created the point *F* that we desired.

Now hide the objects that we constructed in the process of creating line $\overleftrightarrow{CF}$.

38. Hide line $\overleftrightarrow{AF}$, line $\overleftrightarrow{DE}$, point *D*, and point *E*.

Measure stuff

39. Measure $\measuredangle ABC$ and $\measuredangle FCB$.

Wrap up

Your drawing should now show the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CF}$ and transversal $\overleftrightarrow{BC}$. The two angles $\measuredangle FDC$ and $\measuredangle ADB$ are congruent alternate interior angles. Their measurements should be displayed. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CF}$ should look parallel.

40. Put a picture of your drawing in its current state into your MSWord document
    - Save an picture of your NonEuclid workspace by typing [Alt]+[Print Screen]. This saves a picture of the active window on the computer clipboard.
    - Paste the picture that you just saved into your MSWord document (the same document called 330A_CP3_YourLastName).
    - Resize and crop the picture so that it takes up about ¼ page and looks good.
    - Type a a caption and explanation for the picture.
41. Back in your drawing in NonEuclid, move point *C* (drastically)
42. Yell “Ahah!” (Your drawings illustrate the Alternate Interior Angle Theorem: You created Alternate Interior Angles $\measuredangle FDC$ and $\measuredangle ADB$ that are congruent, and as a result, lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CF}$ are parallel. As you move point *C* around, the alternate interior angles remain congruent (because of the way you constructed your drawing) and as a result, the lines remain parallel.)
43. Put a picture of your drawing in its current state into your MSWord document
    - Resize and crop the picture so that it takes up about ¼ page and looks good. (Your two NonEuclid drawings and their captions should fit on one page in MSWord.)
    - Type a a caption and explanation for the picture.

Part II: Drawings to illustrate the Converse of the Alternate Interior Angle Theorem

Geometer's Sketchpad Tasks (Drawings of Euclidean Geometry)

44. Erase your *GSP* drawing.
45. Create a line $\overleftrightarrow{AB}$.
46. Give line $\overleftrightarrow{AB}$ the label *L*.
47. Create another line $\overleftrightarrow{BC}$ that intersects line *L* at point *B*.
48. Give line $\overleftrightarrow{BC}$ the label *T*.

The goal is to create a line *M* that passes through *C* and is parallel to line *L*.

49. With the selection tool (the arrow), unselect everything, then select point *C* and line *L*.
50. On the [Construct] menu, click [parallel line]. *GSP* will create a parallel line without any additional points on it.
51. Give the parallel line the label *M*.
52. On line *M*, create a point *D* that is on the other side of line *T* from point *A*.

Measure stuff

53. Measure $\measuredangle ABC$ and $\measuredangle DCB$.

Wrap up

Your drawing should now show the parallel lines *L* and *M* and transversal *T*. The two angles $\measuredangle DCB$ and $\measuredangle ABC$ are alternate interior angles. Their measurements should be displayed.

54. Save a picture of your drawing in its current state.
    - On the GSP [File] menu, click [Save As...]
    - Save in: CP2 folder that you created on the desktop
    - File name: drawing 3
    - Save as type: Windows Metafile (*.wmf)
55. Back in your drawing in GSP, move point C (drastically).
56. Say "Hmmm!" (Your drawings seem to indicate that the Converse of the Alternate Interior Angle Theorem is true in Euclidean Geometry . You created parallel lines *L* and *M* and a transveral *T*, and the resulting alternate interior angles seem to be congruent.. As you move point *C* around, lines *L* and *M* remain parallel (because of the way you constructed your drawing) and the resulting alternate interior angles seem to remain congruent. Keep in mind, though, that these drawings don't officially prove anything. They are just drawings.)

Save another image of your drawing in its current state.

- File name: drawing 4

Non-Euclid Tasks (Drawings of the Poincare Disk

57. Open a new drawing in *NonEuclid*
58. Create a line $\overleftrightarrow{AB}$.
59. Create another line $\overleftrightarrow{BC}$ that intersects line $\overleftrightarrow{AB}$ at point *B*.
60. Create another line $\overleftrightarrow{CD}$ that intersects line $\overleftrightarrow{BC}$ at point *C*, with the additional property that point *D* is on the opposite side of line $\overleftrightarrow{BC}$ from point *A*.
61. Move point *D* around, if necessary, to a location such that line $\overleftrightarrow{CD}$ does not intersect line $\overleftrightarrow{AB}$.

Measure stuff

62. Measure $\measuredangle ABC$ and $\measuredangle DCB$.

Wrap up

Your drawing should now show the parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ and transversal $\overleftrightarrow{BC}$. The two angles $\measuredangle ABC$ and $\measuredangle DCB$ are alternate interior angles. Their measurements should be displayed.

63. Put a picture of your drawing in its current state into your MSWord document
    - Save an picture of your NonEuclid workspace by typing [Alt]+[Print Screen]. This saves a picture of the active window on the computer clipboard.
    - Paste the picture that you just saved into your MSWord document (the same document called 330A_CP3_YourLastName).
    - Resize and crop the picture so that it takes up about ¼ page and looks good.
    - Type a a caption and explanation for the picture.
64. Back in your drawing in NonEuclid, move point *C* (drastically), making sure that lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ remain parallel (do not intersect.)
65. Say "Hmmm?" (Your drawings seem to indicate that the Converse of the Alternate Interior Angle Theorem is *not* true in the Poincare disk . You created parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ and transversal $\overleftrightarrow{BC}$, and the resulting alternate interior angles were probably not congruent to begin with.. As you move point *C* around, lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ remain parallel (you make sure of this when you move point *C*) but the resulting alternate interior angles are not congruent.
66. Put a picture of your drawing in its current state into your MSWord document

Microsoft Word Tasks

67. Save your MSWord document.

Blackboard Tasks

68. Put your document in the "digital dropbox".

Computer tasks

69. Close GSP
70. Close NonEuclid
71. Close MSWord
72. Logout of Blackboard
73. Logout of the computer