Math 266A (Barsamian) Computer Project 5: Creating Graphs for Exercise 2.2#4

In exercise 8.2#4 from Homework 5, you will investigate the equilibria of the differential equation

$$\frac{dy}{dx} = y(1-y)(y-2)$$

You will make a graph of the function $g(y) = y(1-y)(y-2)$ and use it to find the equilibria and determine whether each equilibrium is stable or unstable. What you don't actually do in that homework problem is solve the differential equation. Remember that a solution of the differential equation would be a function $y(x)$ that makes the differential equation true. Such a function would contain an undetermined constant because of the integration involved in the solution.

Suppose that the differential equation above were part of an initial value problem:

$$\begin{cases} \dfrac{dy}{dx} = y(1-y)(y-2) \\ y(0) = y_0 \end{cases}$$

Where the symbol y_0 represents some given constant that is the initial value. Remember that a solution of the differential equation would be a function $y(x)$ that makes both the differential equation and the initial condition true. Such a function would not contain an undetermined constant.

It should be no surprise that solving the initial value problem above is extremely messy. But it is instructive to discuss the solutions, because they help us understand the idea of equilibria and their stability. I have solved the initial value problem for six different initial values. The resulting six functions are called $y_a(x)$, $y_b(x)$, $y_c(x)$, $y_d(x)$, $y_e(x)$, $y_f(x)$, In this MATLAB assignment, the goal is to make graphs of those six solutions. In making the graphs, you will acquire some important basic MATLAB skills, including:

- defining variables and functions
- graphing a function on a specified domain
- changing the line style of a graph
- using the "up arrow" and "down arrow" keys as a shortcut to typing
- graphing more than one function on the same set of axes
- adjusting the scale of the axes of a graph
- adding a title to a graph

Start the MATLAB program. You should see a mostly-empty workspace with the ">>" symbol at the left. This workspace is called the *command window*, and the ">>" symbol is called the *command prompt*. The prompt indicates that MATLAB is ready to receive input from you. You will type input after the prompt.

You will start by defining a function of one variable, and graphing it.

1) At the *command prompt*, type `clear`enter This erases any previously stored values.

2) At the *command prompt*, type `syms x`enter (Be sure to include the space.) This creates a symbolic variable called *x*.

3) At the *command prompt*, type `ya=0`enter (Be sure to include all characters shown, with correct upper & lower case and no spaces.) This creates a function $y_a(x) = 0$ that is a constant function with value 0.

4) At the *command prompt*, type
`yb=((1+3*exp(-2*x))-sqrt(1+3*exp(-2*x)))/(1+3*exp(-2*x))`enter
(Be sure to include all characters shown, with correct upper & lower case and no spaces.)
This creates a function $y_b(x) = \dfrac{\left(1+3e^{-2x}\right) - \sqrt{1+3e^{-2x}}}{\left(1+3e^{-2x}\right)}$

5) At the *command prompt*, type `yc=1`enter (Be sure to include all characters shown, with correct upper & lower case and no spaces.) This creates a function $y_c(x) = 1$ that is a constant function with value 1.

6) At the *command prompt*, press the up arrow key a couple of times and then press the down arrow key a couple of times. The program will respond by displaying recent lines of text that have been entered. Using the up and down arrow keys as necessary, get the program to display the line of text that you used to create the function $y_b(x)$.
`yb=((1+3*exp(-2*x))-sqrt(1+3*exp(-2*x)))/(1+3*exp(-2*x))`
Modify that line of text to read the following
`yd=((1+3*exp(-2*x))+sqrt(1+3*exp(-2*x)))/(1+3*exp(-2*x))`
(All that you will have to do is change the "b" to "d" and change the "-sqrt" to "+sqrt".
With that new line of modified text displayed, hit enter.
This creates a function $y_d(x) = \dfrac{\left(1+3e^{-2x}\right) + \sqrt{1+3e^{-2x}}}{\left(1+3e^{-2x}\right)}$

7) At the *command prompt*, type `ye=2`enter (Be sure to include all characters shown, with correct upper & lower case and no spaces.) This creates a function $y_e(x) = 2$ that is a constant function with value 2.

8) At the *command prompt*, use the up and down arrow keys as necessary, get the program to display the line of text that you used to create the function $y_d(x)$.
`yd=((1+3*exp(-2*x))+sqrt(1+3*exp(-2*x)))/(1+3*exp(-2*x))`
Modify that line of text to read the following
`yf=((1-(5/9)*exp(-2*x))+sqrt(1-(5/9)*exp(-2*x)))/(1-(5/9)*exp(-2*x))`
With that new line of modified text displayed, hit enter.
This creates a function $y_f(x) = \dfrac{\left(1-(5/9)e^{-2x}\right) + \sqrt{1-(5/9)e^{-2x}}}{\left(1-(5/9)e^{-2x}\right)}$

9) At the *command prompt*, type `ezplot(ya,0,3)`enter This command tells MATLAB to graph the function *ya* on the domain $0 \le x \le 3$. The graph will pop-up in a new window entitled *Figure 100*, or something similar.

Now you will change the solid graph to a dotted graph.

10) Across the top of the graph's window, you will see the names of various available pull-down menus: *File*, *Edit*, *View*, *Insert*, etc. Below this row of words, you should see a row of symbols. This row of symbols is called the *Figure Toolbar*. (If you don't see the *Figure Toolbar*, then click on the word *View* to pull down the *View menu*, and on that menu, click on the words *Figure Toolbar*. This should put a check next to those words, causing the *Figure Toolbar* to appear.) One of the symbols on the *Figure Toolbar* is a white arrow. Click once on the white arrow. This tells MATLAB that you want to edit the plot. (Repeated clicks on this white arrow will switch you in and out of the *plot editing mode*. The MATLAB program indicates that you are in *plot editing mode* by showing a little rectangle around the white arrow on the *Figure Toolbar*.)

11) With your mouse, move the white arrow so that it touches the line of the graph, and click on the graph. A bunch of squares or dots should appear on the graph, indicating that you are now editing the graph.

12) At the bottom of the graph window, a smaller window should appear, entitled *Property Editor – Lineseries*. (If it does not, then double-click on the graph.) Within this window, there should be a smaller window titled *line*. Clicking on the little triangle next to this window will enable you to change the line style for the graph that you are editing. Change the line style to make the graph a dotted line.

13) You are done with the graph window for now. Click somewhere in the blank space of the *command window* in order to bring that window to the front.

Now you will create five more graphs on the same set of axes.

14) First you need to tell MATLAB not to erase the graph that you just made. At the *command prompt*, type `hold on`enter This command tells MATLAB to not erase old graphs when a new graph is created.

15) At the *command prompt*, type `ezplot(yb,0,3)`enter The Figure 100 window will pop-up again, this time showing two graphs. The old graph is dotted or dashed; the new graph is solid.

16) Return to the *command window*. You do not need to enter the *hold on* command again.

17) At the *command prompt*, type `ezplot(yc,0,3)`enter The Figure 100 window will pop-up again, showing the new graph. The earlier two graphs might not be visible, depending on the scales of the axes. But they have not gone away. In a later step, you will adjust the axes and then you will see the old graphs again.

18) Return to the *command window*. You do not need to enter the *hold on* command again.

19) At the *command prompt*, type `ezplot(yd,0,3)`enter The Figure 100 window will pop-up again, showing the new graph as a solid line. Leave this graph as a solid line.

20) At the *command prompt*, type `ezplot(ye,0,3)`enter The Figure 100 window will pop-up again, showing the new graph. The earlier two graphs might not be visible, depending on the scales of the axes. But they have not gone away. In a later step, you will adjust the axes and then you will see the old graphs again.

21) Return to the *command window*. You do not need to enter the *hold on* command again.

22) At the *command prompt*, type `ezplot(yf,0,3)`enter The Figure 100 window will pop-up again, showing the new graph as a solid line. Leave this graph as a solid line.

Now you will adjust the axes.

23) Be sure that you are in the *plot editing mode*. (Click on the white arrow, if necessary.)

24) Double click somewhere on the axes of the graph. The window below the graph should change to one entitled *Property Editor – Axes*.

25) In the window titled *Property Editor - Axes*, click on the tab called *Y axis*. This will open a smaller window. In this smaller window, there will be a small box titled *auto*. If there is a check in this box, click on the box to make the check go away. (You *do not* want the *y*-axis to be automatically scaled.)

26) Enter Y-limits of -0.5 to 3 in the boxes provided. (After you type a number in the box, you will need to either press enter or click with your mouse somewhere outside the box, in order to submit the typed value.) With this change in the scale of the axes, all four graphs should now be visible.

Now you will change two more solid graphs to dotted graphs.

27) Change the line style of the graphs of yc and ye to also be dashed. (Use the method of steps 10 - 13.)

Now you will add a title to the graph.

28) In the Figure 100 window, above the graph, you should see a title consisting the formula for the last graph that you created. The MATLAB program has just used this formula for the most recent graph as the title for the whole picture. That is not very helpful. Your next chore is to make a better title. In the edit plot mode, double click on the existing title The program will highlight the title by putting little black boxes around it, and the area around the formula will turn white. Erase the formula and enter the following two-line title in its place:

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Six solutions of the Diff EQ dy/dx = y(1-y)(y-2) for exercise 8.2#4
your name, Math 266A Computer Project 5, due Friday February 22, 2008
```

Finishing up.

29) Your figure is now complete. Print it. (Click on the word *File* to pull down the *File* menu, and on that menu, click on the *Print...* symbol.)

30) Exit the MATLAB program. (Click on the word *File* to pull down the *File* menu, and at the bottom of that menu, click on *Exit Matlab*.)

31) Use the figure to illustrate your solution to exercise 8.2#4.