

Limit Laws from Section 1.4

Suppose that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist. Then

Limit Law 1:	$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
Limit Law 2:	$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
Limit Law 3:	If c is a constant, then $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$
Limit Law 4:	$\lim_{x \rightarrow a} [(f(x))(g(x))] = \left(\lim_{x \rightarrow a} f(x)\right)\left(\lim_{x \rightarrow a} g(x)\right)$
Limit Law 5:	If $\lim_{x \rightarrow a} g(x) \neq 0$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$
Limit Law 6:	$\lim_{x \rightarrow a} [(f(x))^n] = \left[\lim_{x \rightarrow a} f(x)\right]^n$ where n is a positive integer.
Limit Law 7:	If c is a constant, then $\lim_{x \rightarrow a} c = c$
Limit Law 8:	$\lim_{x \rightarrow a} x = a$
Limit Law 9:	If n is a positive integer, then $\lim_{x \rightarrow a} x^n = a^n$.
Limit Law 10:	<ul style="list-style-type: none"> • If n is a positive odd integer, then $\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$ • If n is a positive even integer and $a > 0$, then $\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$
Limit Law 11:	<ul style="list-style-type: none"> • If n is a positive odd integer, then $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(a)}$ • If n is a positive even integer and $\lim_{x \rightarrow a} f(a) > 0$, then $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(a)}$
Direct Substitution Property:	<ul style="list-style-type: none"> • If f is a polynomial, then $\lim_{x \rightarrow a} f(x) = f(a)$. • If f is a rational function and a is in the domain of f, then $\lim_{x \rightarrow a} f(x) = f(a)$.
Fact from p. 38:	<p>If $f(x) = g(x)$ for all x such that $x \neq a$, then $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ will agree. That is,</p> <ul style="list-style-type: none"> • If one of the limits exists and has value L, then the other limit exists and has value L. • If one of the limits does not exist, then the other limit does not exist.
Theorem 2:	$\lim_{x \rightarrow a} f(x) = L$ if and only if $\left(\lim_{x \rightarrow a^-} f(x) = L\right) \text{ AND } \left(\lim_{x \rightarrow a^+} f(x) = L\right)$.
Theorem 3:	If $f(x) \leq g(x)$ when x is near a (except possibly at a), and if both $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist, then $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$.
Squeeze Theorem:	If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a), and if both $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} g(x) = L$.
Fact #5 from p. 42:	$\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1$