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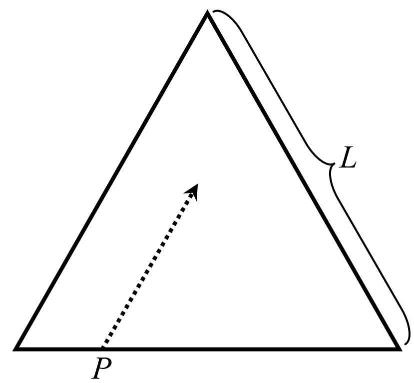
Geometry In-Class Exam 2
Math 330B/539 (Barsamian)
Friday, February 26, 2009
No Calculators

Problem:	1	2	3	4	5	6	Total
Score:							
Possible:	50	40	40	40	40	40	250

[1] Prove that if a map $f:\mathbb{R}^2 \to \mathbb{R}^2$ is distance-preserving then it is also one-to-one and onto.

[2] (Suggested problem 5.2#3) Three mirrors are arranged in an equilateral triangle with sides of length L. From a point P on one of the mirrors, a beam of light is sent in a direction parallel to one of the other mirrors. Eventually the beam will return to point P and begin repeating its journey along the same path.



(a) Describe the path that the beam of light follows. There are two cases to describe:

- Case 1: point P is a midpoint.
- Case 2: point P is not a midpoint.

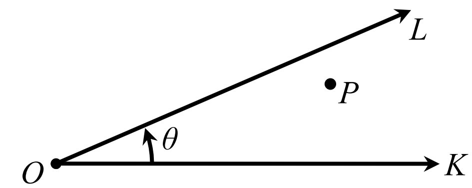
(b) In the two cases above, determine the length of the path in terms of L. Explain how you got your answer. (No credit for memorized answers.)
Hint: This problem can be solved using techniques from Section 5.2, but it can also be solved by other methods. The methods used to solve one of your homework problems can be applied.

[3] (Suggested problem 5.3#5) Given a convex quadrilateral($ABCD$), prove that the point P at the intersection of the diagonals is the minimum distance point for the quadrilateral. That is, point P is the point from which the sum of the distances to the vertices is minimal.
Hint: Let Q be some other point and compare the sum $QA + QB + QC + QD$ to the sum $PA + PB + PC + PD$ by applying the triangle inequality twice.

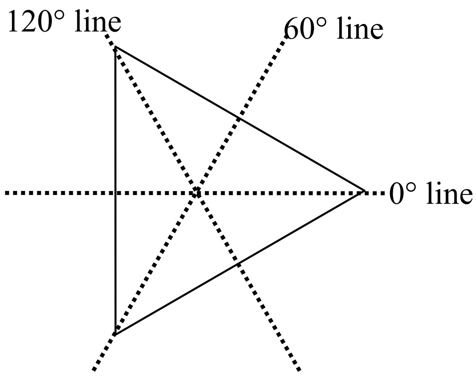
[4] Create a Geogebra drawing for the previous problem. Instructions:

- Points A, B, C, D, and Q should be free; Point P should be dependent.
- Points and segments should be named as in the problem statement..
- An object $SUM\_P = PA + PB + PC + PD$ should be created, and an analogous one $SUM\_Q = QA + QB + QC + QD$. These two objects should appear in the list of objects.
- Your name should appear in a text box somewhere in the drawing.
- Your file should be saved with the filename M2.Lastname
- Don't ZIP or compress your file!!
- e-mail me your file using your OU e-mail account:
    - recipients:
        - me: Mark.Barsamian.1@ohio.edu
        - you: your OU e-mail address
    - subject line: M2.Lastname
    - attachment: your file called M2.Lastname (Don't ZIP or compress it!)

[5] (Suggested problem 6.1#7) Let point P be in the interior of angle θ formed by the lines K and L as shown. Find the points P' and P'' defined by $P \xrightarrow{M_K} P' \xrightarrow{M_L} P''$. Show that in this case, $m(\angle POP'') = 2\theta$. (This shows that if P is in the interior of θ, then P'' can be obtained from P by a rotation $R_{O,2\theta}$.)



[6] In this problem, you will explore the *dihedral group with eight elements*, D_6. This is the group of symmetries of an equilateral triangle. If we orient the triangle with its center at the origin, and one of the vertices lying on the positive *x-axis*, as shown in the figure below, then the elements of this group are as shown in the list to the right.



Elements of the group D_6
R_{0° = identity = id
R_{120° = counterclockwise rotation through 120°
R_{240° = counterclockwise rotation through 240°
M_1 = reflection in the 0° line
M_2 = reflection in the 60° line
M_3 = reflection in the 120° line
(In the figure, you should think of the degree lines as being fixed to the table, while the square is the thing that gets rotated and flipped. For instance, the 0° line is always horizontal, and the 60° line always goes from lower left to upper right, as shown.)

Your task is to fill out the group table below

	$\circ$	transformation to the right of ∘ symbol					
		id	R_{120°	R_{240°	M_1	M_2	M_3
transformation to the left of ∘ symbol	id						
	R_{120°						
	R_{240°						
	M_1						
	M_2						
	M_3						