

## Transformations, Progress Report 1

### Fixed points and fixed lines

**Introduction and instructions:** This project will investigate the relation between the properties of reflections and the parallel postulate assumed. As a warm-up activity, this progress report asks you to examine reflections on the Lenart sphere with respect to fixed points and lines (defined below).

**Definition 1** *Let  $M$  be a mapping from the plane to itself. A point  $P$  is called a fixed point of  $M$  if  $M(P) = P$  and a line  $b$  is called a fixed line of  $M$  if  $M(b) = b$ .*

1. Define a mapping on the Lenart sphere that satisfies all of the criteria for a reflection and M.1 and M.2.
2. Is every point of a fixed line a fixed point? Justify your answer.
3. Find all of the fixed points and fixed lines for Mira/GSP reflections and for reflections on the Lenart sphere.
4. Using only the provided axioms and definitions, give complete proofs of the following propositions:
  - (a) If  $P$  and  $Q$  are fixed points of  $R_a$ , then the line determined by  $P$  and  $Q$  is a fixed line of  $R_a$ .
  - (b) If  $l$  and  $m$  intersect at a point  $P$  and are different fixed lines of a reflection  $R_a$ , then  $P$  is a fixed point of  $R_a$ .
  - (c) Every perpendicular of a line  $a$  is fixed by the reflection  $R_a$ .
5. Which member of your group travelled the farthest from Athens over Spring break? How far did they go?