Progress Report 3

Half turns

Introduction and instructions

Definition 1 *A half turn about a point P is a rotation $R_m R_l$ where the axes of the reflections l and m are perpendicular and intersect at P.*

If H_P is a half-turn, then *there is* a pair of perpendicular lines l and m that meet at P such that $H_P = R_m R_l$. The definition does not state that $H_P = R_u R_v$ for any pair of perpendicular lines that meet at P; this, however, is the conclusion of the first problem. In answering questions that ask you to either "support" or "justify" an assertion, provide a summary version of the proof. Your answer should make it clear how you would go about proving the assertion and include any key observations necessary for your proof. Also, your argument should only rely on the axioms of transformational geometry or there consequences.

Questions:

1. Let P be a point and H_P a half-turn about P and suppose that u and v are perpendicular and meet at P. Justify the following assertion: $H_P = R_u R_v$. (This shows that a half-turn is independent of the choice of the pair of perpendicular lines used to represent it.)

2. Make conjectures regarding the fixed lines of a half-turn in a.) a nonelliptic plane and in b.) an elliptic plane. Prove your conjectures.

3. Sometimes a half turn H_P is called a reflection through P . Explain why. Your explanation should include some analysis of the action of a half-turn.

4. Prove the following: Let P be a point and l be a line. $H_P = R_l$ if and only if P is a pole of l. [Hint: Show that if P is a pole of l, then H_P satisfies all of the criteria of a reflection. In particular, show that H_P fixes every point of l. To prove the converse, show that if $H_P = R_u R_v = R_l$, then u and v must be perpendicular to l.]

5. Justify: The only fixed points of a half-turn about a point P are: a.) P and all the points of the polar line of P if the plane is elliptic and b.) only P if the plane is not elliptic.

6. My favorite group process question: What is your favorite math joke?