

Progress Report #5.4

Products of four or reflections

Introduction and instructions: As the title suggests, this project looks at the product of four or reflections. Such a product can always be reduced to a product of three or fewer reflections.

Questions:

1. Any motion of the form $H_P R_l R_m$ can be replaced by the product of two reflections. (i.e., two reflections followed by a half-turn can be replaced by the product of two reflections.) Prove this for the cases that the lines m and l intersect and that the lines l and m have a common perpendicular. The result is still true if the lines do not intersect and that they do not have a common perpendicular; do not worry about this case. One way to prove this result is to represent the half-turn H_P as a product $R_u R_v$ where v , l , and m have the property that $R_v R_l R_m$ can be replaced by a single reflection.
2. Prove: Any product of the form $R_n R_m R_l$ can be replaced by a product of the form $H_P R_u$. (i.e., three reflections can always be replaced by a reflection followed by a half turn.) One way to approach this problem in the case that l and m intersect is to insert the identity between R_n and R_m in the form $R_u R_u$ where $R_n R_u$ forms a half turn.
3. Justify: The above two observations provide the means of reducing any product of reflections to a product of three or less.