

## Study Guide - Exam 2

The exam will be on Wednesday, June 2 and cover the material starting with progress report 5.4 and ending with the Ruler Postulate. The format will be similar to the previous exam: some 'circle the best answers', support the statement, prove some statements you should be familiar with, and perhaps a couple of new things to consider. You will have the full lab period to complete the exam. As you can tell from the list, we have covered quite a variety of material! I have listed some of the highlights that you should be familiar with.

### **From the axiomatic approach to transformational geometry:**

- Show that any motion of the form  $H_P R_l R_m$  is a quasi-rotation. Be able to use the Join Theorem to establish this result. (This, along with other material related to the final report, will be posted Wednesday afternoon.)
- Be able to show that if the point  $P$  is not on the line  $a$ , then  $H_P R_a$  is a glide reflection.
- Given a sketch with the lines  $l$ ,  $m$ , and  $n$ , be able to find the approximate location of the point  $P$  and the line  $a$  such that motion  $R_l R_m R_n = H_P R_a$
- Use an induction argument to show that the composition of finitely many reflections can be reduced to three or less.
- Be able to use the Euclidean Parallel Postulate in tandem with the axioms of transformational geometry to show that - in the Euclidean plane - if two lines are parallel, then they admit a common perpendicular.
- Be able to answer questions similar to the ones that appeared on the second homework assignment (putt-putt golf and symmetries of a figure.)
- Prove that every motion in an elliptic plane is a rotation.
- Be able to describe the fixed points and fixed lines of any motion in the Euclidean plane. (This is great material for 'circle the best answer' questions.)

**Motions in the Euclidean and plane and related material:**

- Be able to prove the Pythagorean theorem using similar triangles.
- Know the definitions of sine and cosine and be able to prove the law of sines.
- Be able state the 'vector' definitions of a reflection with axis  $l$  and a translation.
- Be able to show that if  $l$  and  $m$  are parallel lines, then there is a vector  $\vec{v}$  such that  $R_m R_l = T_{\vec{v}}$ .
- Know the matrix representations of a rotation (of  $\theta$  degrees) centered at the origin and the reflection  $R_l$  when  $l$  is a line passing through the origin that makes an angle of  $\theta$  degrees with the origin.
- Be able to derive the matrix representation of a rotation of  $\theta$  degrees when the rotation is centered at the origin.
- Be able to prove the Ruler Placement Theorem. Know the definitions of function, one-to-one, and onto.
- Given the equation of a line  $l$  and one of the distance functions  $d_1$ ,  $d_2$  or  $d_\infty$ , be able to produce a coordinate system for  $l$ . (i.e., a function from  $l$  to  $\mathbf{R}$  that satisfies the Ruler Postulate.)