

Study Guide for the Final Exam Math 330B, Spring, 2004

The final exam will be similar to the preceding exams, only a page or two longer. There will be some that just ask you to state a definition, some that ask you to provide the reason(s) for each step of an argument, some “circle the best answer” questions and, of course, some of the proofs we have done in class. The ‘circle the best answer’ questions may be based on anything that we have discussed this quarter. There may also be a couple things that you haven’t seen; these, however, will be closely related to the material we have worked on in class.

Definitions: Fixed point, fixed line, rotation, translation, pole, half-turn, glide reflection, and pencil. Know the definitions of sine, cosine and the vector definitions of translation and reflection.

Know how to **prove the following theorems** using the axioms of transformational geometry

- If P and Q are fixed points of the reflection R_a , then the line determined by P and Q is fixed by R_a .
- Every line perpendicular to the line l is fixed by the reflection R_l .
- Let l and m be lines. Then $R_l R_m R_l = R_{m'}$ if and only if $R_l(m) = m'$.
- Let l and m be two distinct lines. Then l is perpendicular to m if and only if $R_l R_m = R_m R_l$.
- If P is a pole of l and m is perpendicular to l , then P is on m .
- If P is a pole of l and P is on a line m , then l is perpendicular to m .
- (First theorem on three reflections). If the lines l , m , and n intersect at a point P , then there is a line w passing through P such that $R_n R_m R_l = R_w$.
- If $M = R_l R_m$ is a rotation about P , then given any line u passing through P there is another line v passing through P such that $M = R_v R_u$.

- If H_P is a half-turn, then $H_P = R_v R_u$ for any pair of perpendicular lines u and v that meet at P .
- Every line passing through P is a fixed line under H_P .
- Every product $H_P R_v R_u$ can be replaced by a product $R_l R_m$.
- In an elliptic plane, every motion is a rotation.
- In a Euclidean plane, every proper motion is either a translation or a rotation; every improper motion is a glide reflection or a reflection.

Also be able to -

- Answer questions similar to the ones that appeared on the second homework assignment (putt-putt golf and symmetries of a figure.)
- Describe the fixed points and fixed lines of any motion in the Euclidean plane.
- Show that if l and m are parallel lines, then there is a vector \vec{v} such that $R_m R_l = T_{\vec{v}}$.
- Use the matrix representations of a rotation (of θ degrees) centered at the origin and the reflection R_l when l is a line passing through the origin that makes an angle of θ degrees with the origin. Be able to derive the matrix representation of a rotation of θ degrees when the rotation is centered at the origin.
- Prove the Ruler Placement Theorem. Know the definitions of function, one-to-one, and onto.
- Produce a coordinate system for a given line l using the equation of a line l and any of the distance functions d_1 , d_2 or d_∞ .