

## Notes for Homework #3

The definition of similar triangles follows the same format as the definition of congruent triangles. One first defines what it means for a correspondence between the vertices of the triangles to be a 'similarity' and then defines two triangles to be similar if there is a similarity between their vertices.

**Definition:** Let  $\triangle ABC$  and  $\triangle DEF$  be two triangles. The correspondence  $ABC \leftrightarrow DEF$  is a similarity provided that corresponding angles are congruent and

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}.$$

In the event the correspondence  $ABC \leftrightarrow DEF$  is a similarity, we write  $\triangle ABC \sim \triangle DEF$  and say that the triangles  $\triangle ABC$  and  $\triangle DEF$  are similar.

Recall that when establishing two triangles are congruent one does not need to establish that every pair of corresponding angles are congruent and every pair of corresponding sides are congruent. The *ASA*, *SAS*, *AAS* and *SSS* criteria for congruence tells us that we just need to test some of the pairs in order to conclude that the two triangles are congruent. The situation is similar for similar triangles. Just as in the case for the congruence of triangles, it turns out that in order show that  $\triangle ABC$  is similar to  $\triangle DEF$ , one does not need to establish that every pair of corresponding angles are congruent and that every pair of corresponding sides are proportional. In particular, one can establish the following:

**Theorem:** Let  $\triangle ABC$  and  $\triangle DEF$  be two triangles. Then:

1. AAA criteria: If the corresponding angles are congruent, then  $\triangle ABC$  is similar to  $\triangle DEF$ .
2. SSS criteria: If  $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ , then  $\triangle ABC$  is similar to  $\triangle DEF$ .
3. SAS criteria: If  $\angle A$  is congruent to  $\angle D$  and  $\frac{AB}{DE} = \frac{AC}{DF}$ , then  $\triangle ABC$  is similar to  $\triangle DEF$ .

During the lecture we also gave a definition of the sine and cosine of an angle. Just for completeness sake, it is included here. It may be useful in problem 2.

**Definition:** Given  $\angle A$ , define  $\sin(\angle A)$  and  $\cos(\angle A)$  as follows:

1. If  $\angle A$  is acute, form a right triangle  $\triangle ABC$  with right angle at  $C$ . Then

$$\sin(\angle A) = \frac{BC}{AB} \text{ and } \cos(\angle A) = \frac{AC}{AB}.$$

2. If  $\angle A$  is a right angle, let  $\sin(\angle A) = 1$  and  $\cos(\angle A) = 0$ .
3. If  $\angle A$  is obtuse and  $\angle A'$  is a supplement of  $\angle A$ , then let  $\sin(\angle A) = \sin(\angle A')$  and  $\cos(\angle A) = -\cos(\angle A')$ .

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Math 330B - Homework 3

**Problem 1:** Using the SAS criteria for similarity (see the first page), prove the ‘midsegment theorem’:  
Let  $\triangle ABC$  be a triangle and suppose that  $D$  is the midpoint of  $\overline{AB}$  and  $E$  is the midpoint of  $\overline{BC}$ . Prove that  $\overleftrightarrow{DE}$  is parallel to  $\overleftrightarrow{AC}$  and  $DE = \frac{1}{2}AC$ . (8 pts.)

**Problem 2** :Prove the *Law of Sines*: If  $\triangle ABC$  is any triangle, then

$$\frac{AB}{AC} = \frac{\sin(\angle C)}{\sin(\angle B)}$$

In order to prove this, let  $D$  be the foot of the perpendicular dropped from  $A$  to  $\overleftrightarrow{CB}$ . The key to the proof is to use compute  $AD$  using  $\sin(\angle C)$  and  $AC$ , compute  $AD$  using  $\sin(\angle B)$  and  $AB$ , set the results equal to one another, and then simplify. Notice that there are a number of cases that need to be considered:  $B - C - D$ ,  $C - D - B$ ,  $D = B$ ,  $D = C$  and  $D - B - C$ .

a. Prove the result when  $C - D - B$ . Include a diagram that illustrates this case. (4 pts.)

b. Prove the result when  $C - B - D$ . Include a diagram that illustrates this case.(4 pts.)